The Fast Fourier Transform on GPUs

Some images in this part by Mario Garrido

The Fast Fourier Transform (FFT)

Fast implementation of the Fourier Transform

Converts a signal to frequency space

Very important algorithm in signal processing

FFT

Computes the Discrete Fourier Transform (DFT) of a signal of N samples in NlogN time

Many variants. Cooley-Tukey (1965) most common.

DFT

The Discrete Fourier Transform

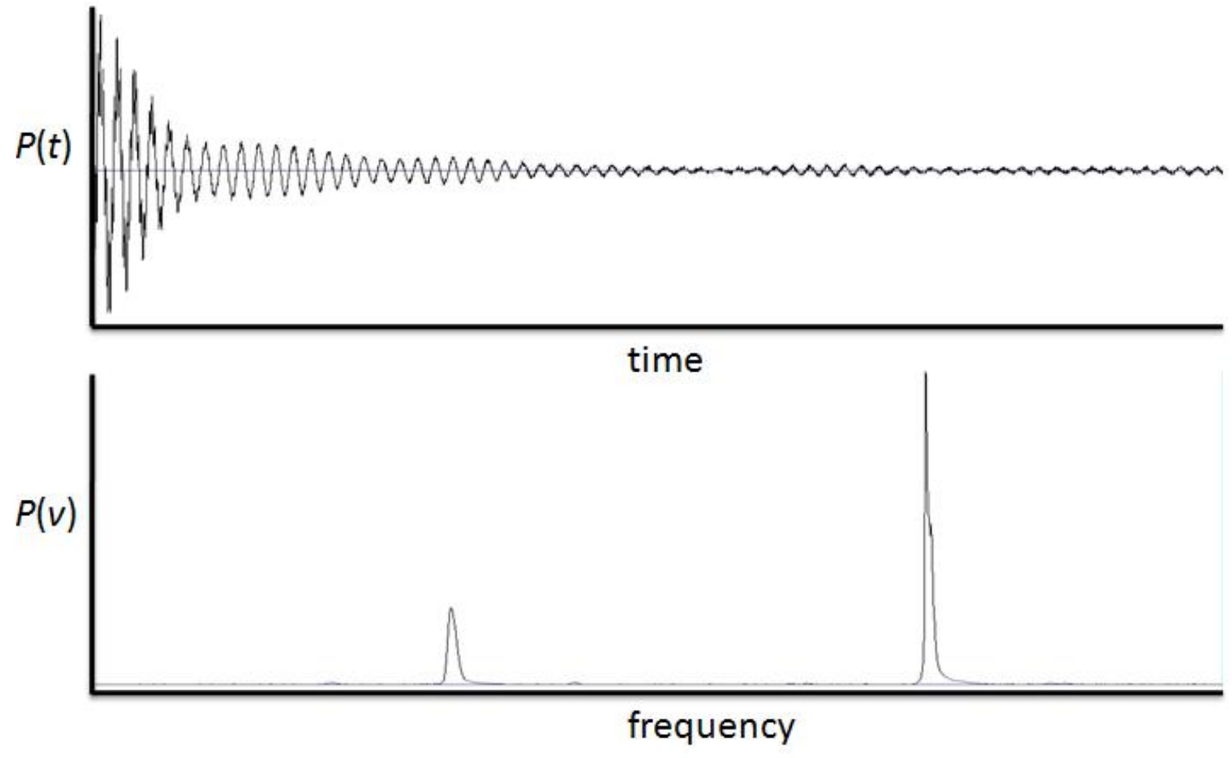
Converts a signal to frequency space

Essentially a series of convolutions with harmonic functions of varying frequency

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i 2\pi k \frac{n}{N}} \qquad k = 0, \ldots, N-1.$$

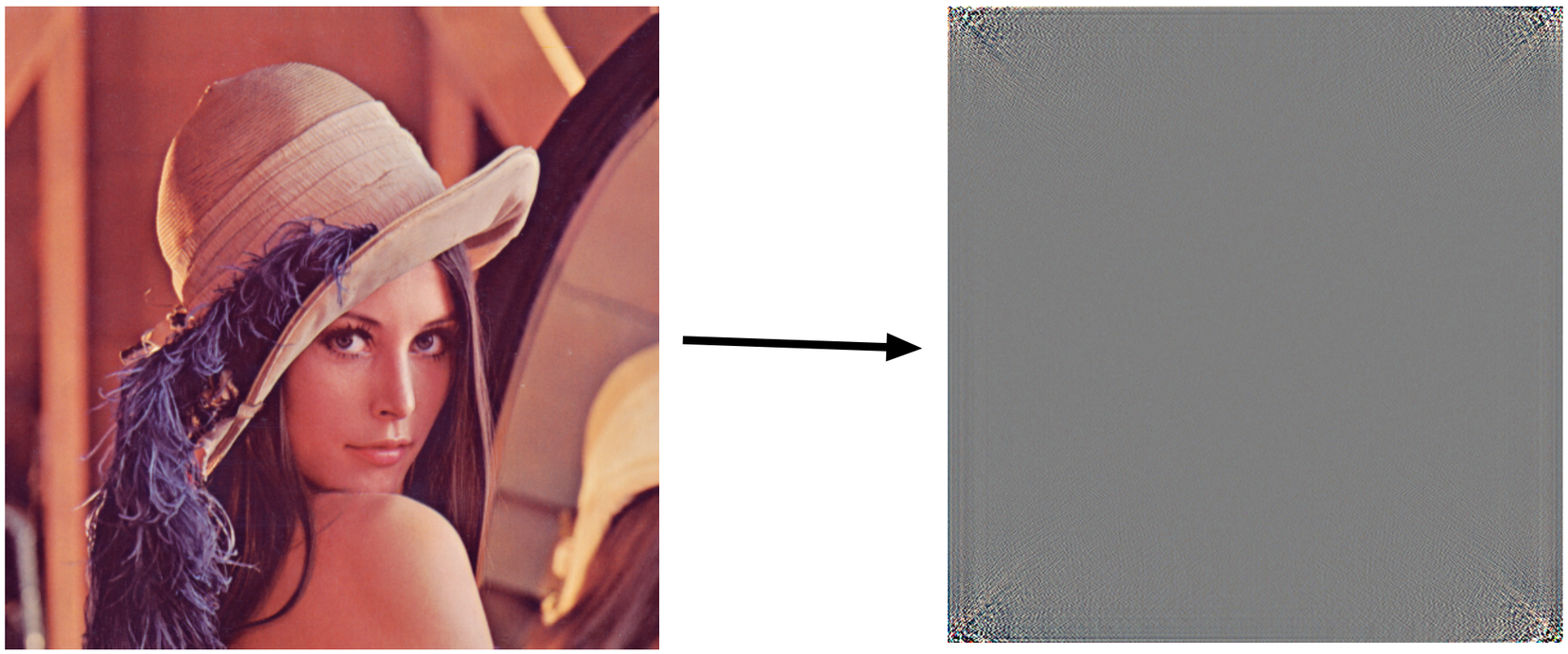
DFT example

1D signal to frequency space (e.g. sound)



DFT example

2D signal to frequency space (e.g. images)



FFT flow graph (Radix-2)

x[n]

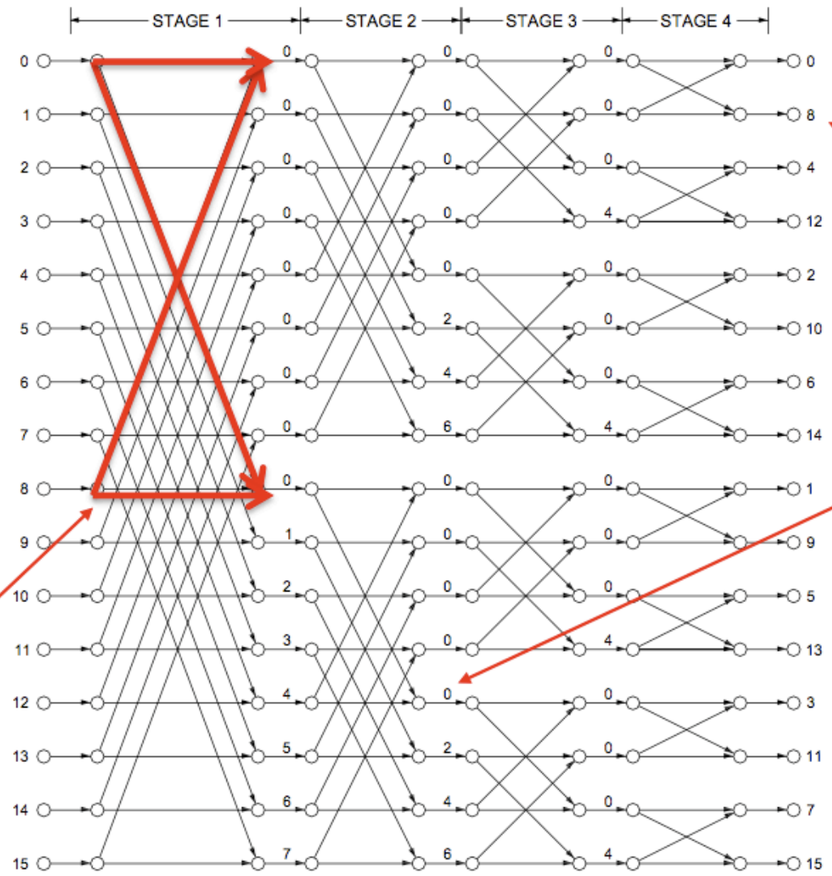
n

N points

$\log_r N$ stages

butterflies

(radix-2)



X[k]

k

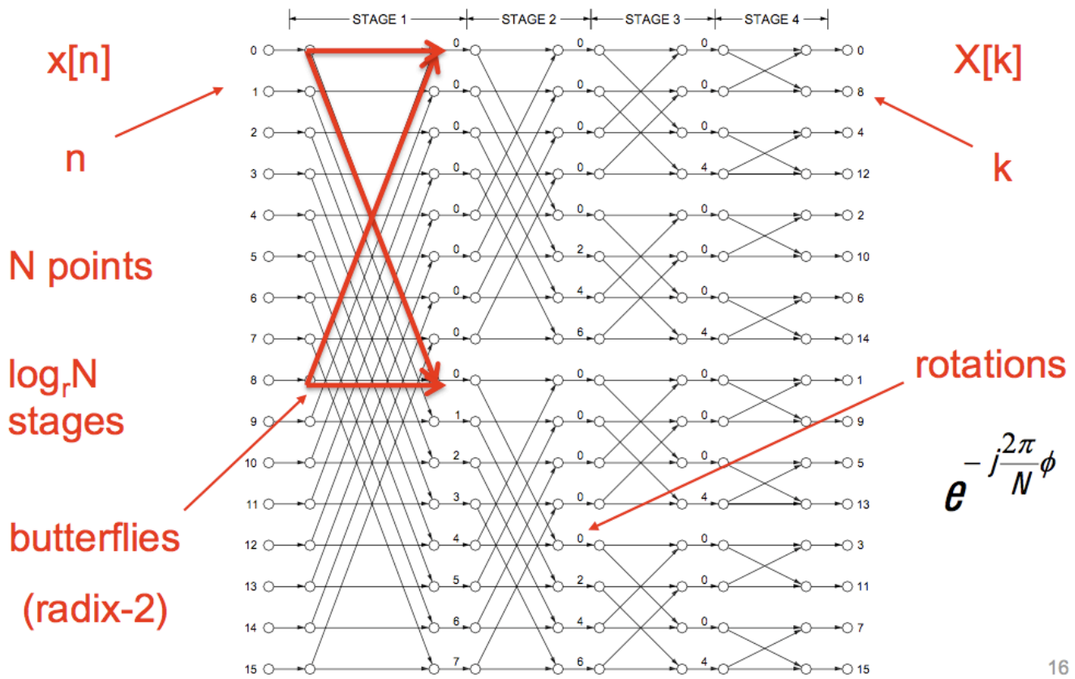
rotations

$$e^{-j\frac{2\pi}{N}\phi}$$

FFT in parallel

Pretty parallel from the start!

BUT very large jumps in memory for some stages!



”Large” stages: Can not be performed within shared memory!

”Small” stages: Can be performed within shared memory!

Possible approach

Perform all ”small” stages in a single run, using shared memory. Very fast!

Perform all ”large” stages as separate kernel runs.

NVidia ”made your bed” for FFT

cufft, CUDA FFT, included in all CUDA distributions

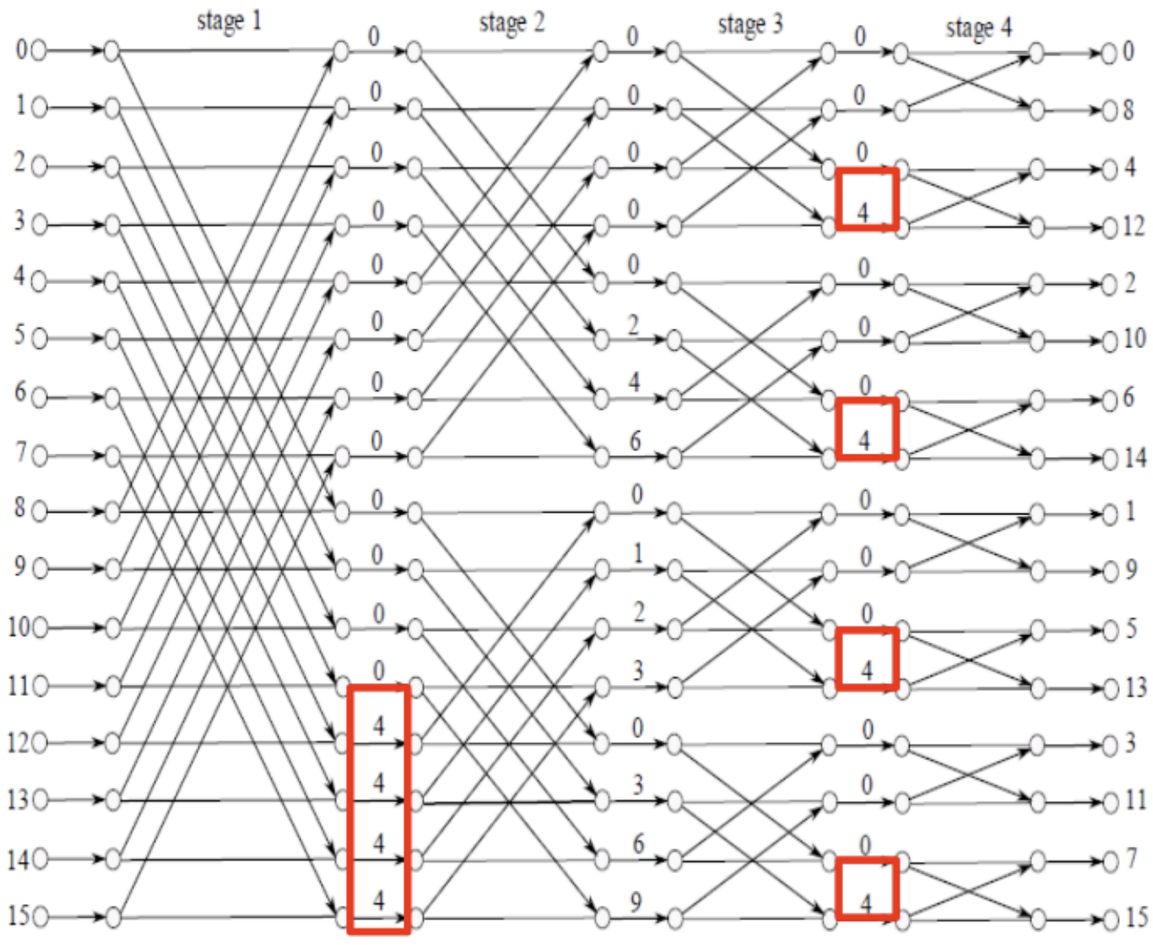
A well optimized CUDA implementation

But there are alternatives!

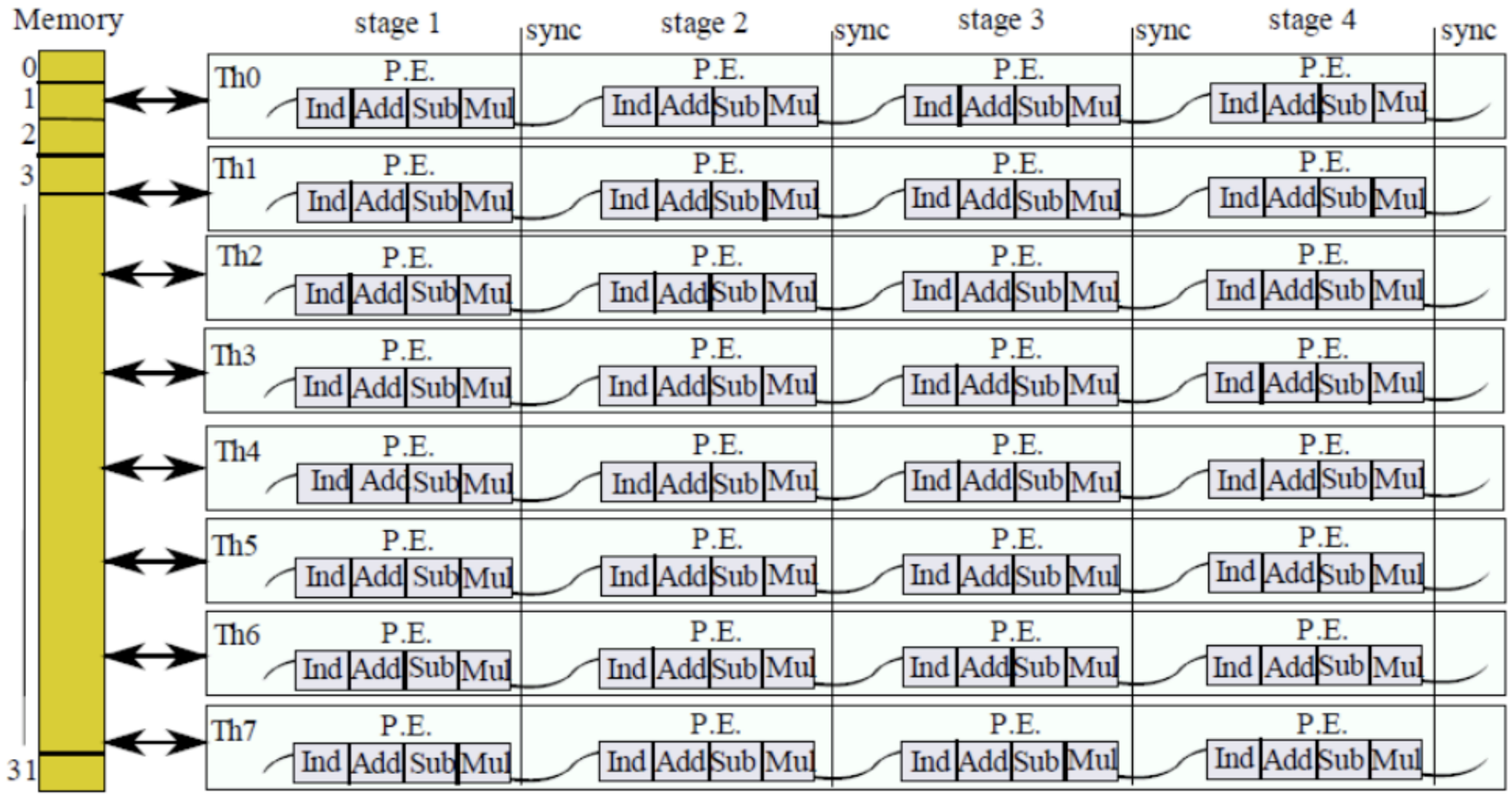
Optimization approaches made in a specific implementation

2013 publication: ”New Radix-2 and Radix-2^2 Constant Geometry Fast Fourier Transform Lagorithms for GPUs”, Ambuluri,Garrido,Ogniewski,Ragnemalm,Caffarena

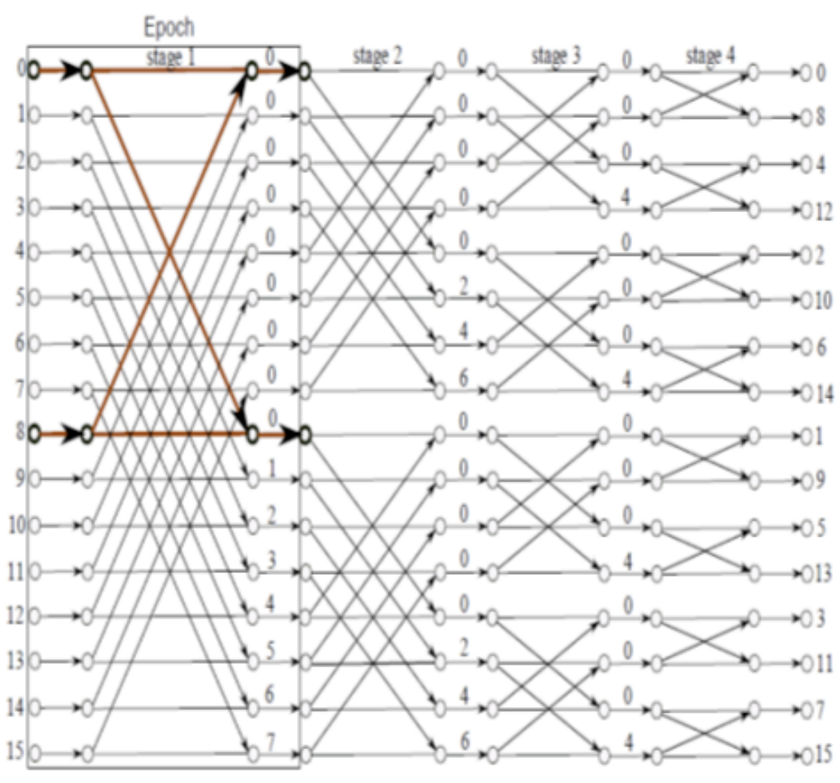
Simplify - use Radix-2^2



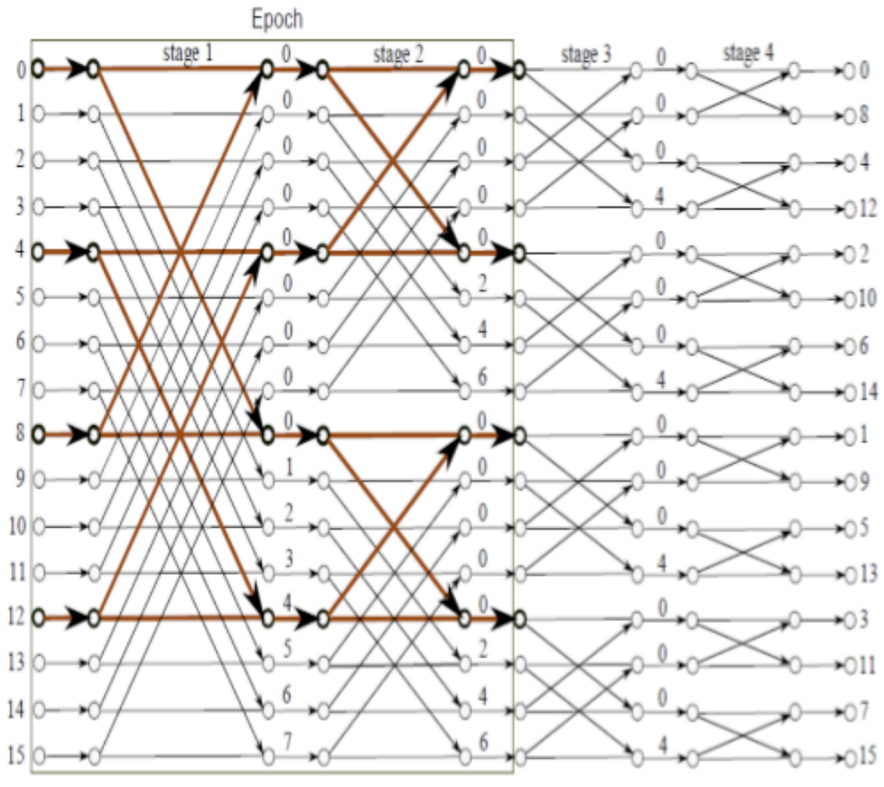
Use shared memory



Reduce synch. points

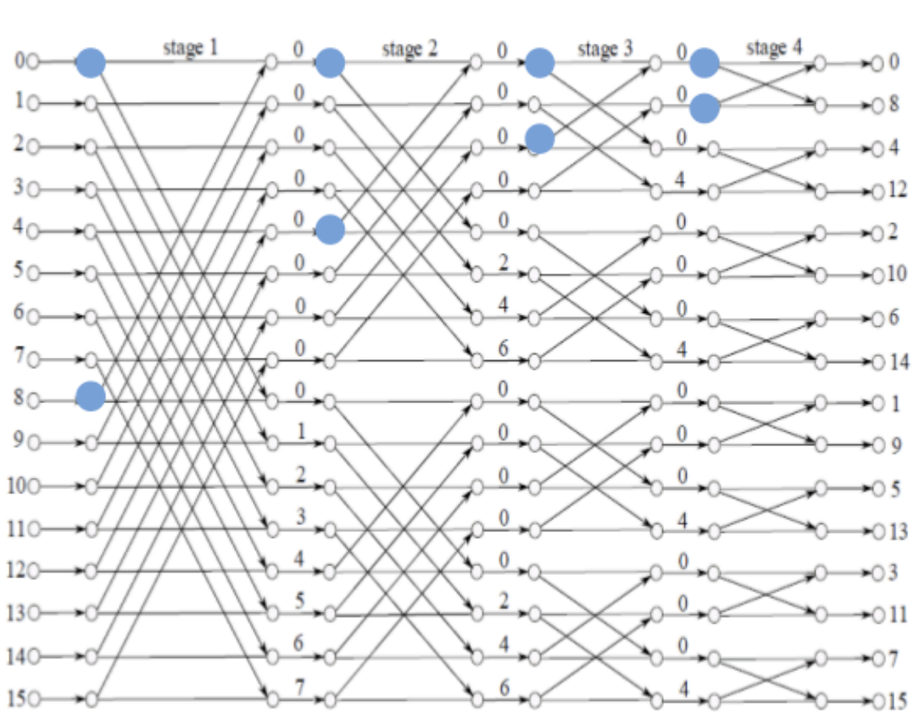


2-word group

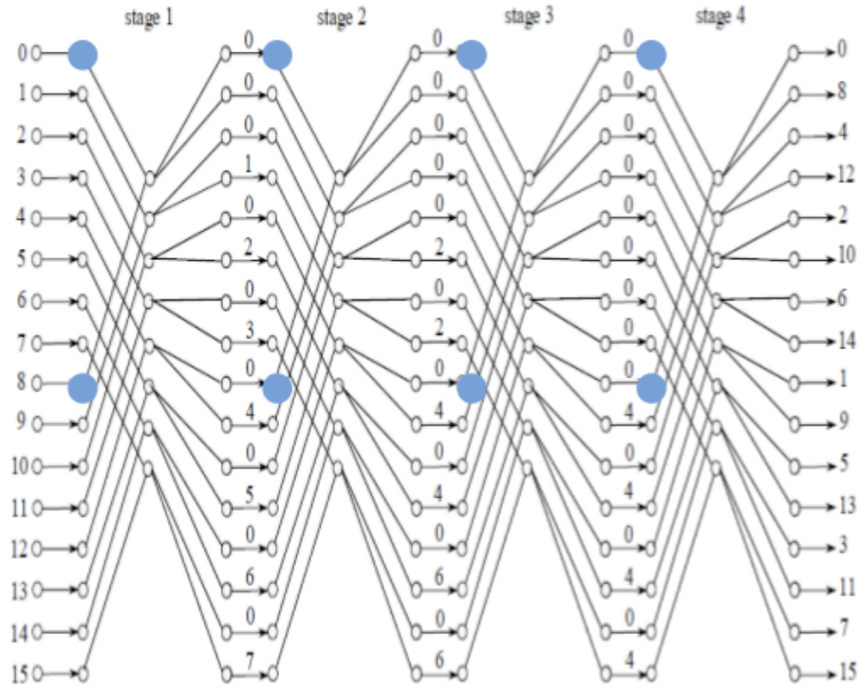


4-word group

Reduce index calculations: Constant geometry FFT

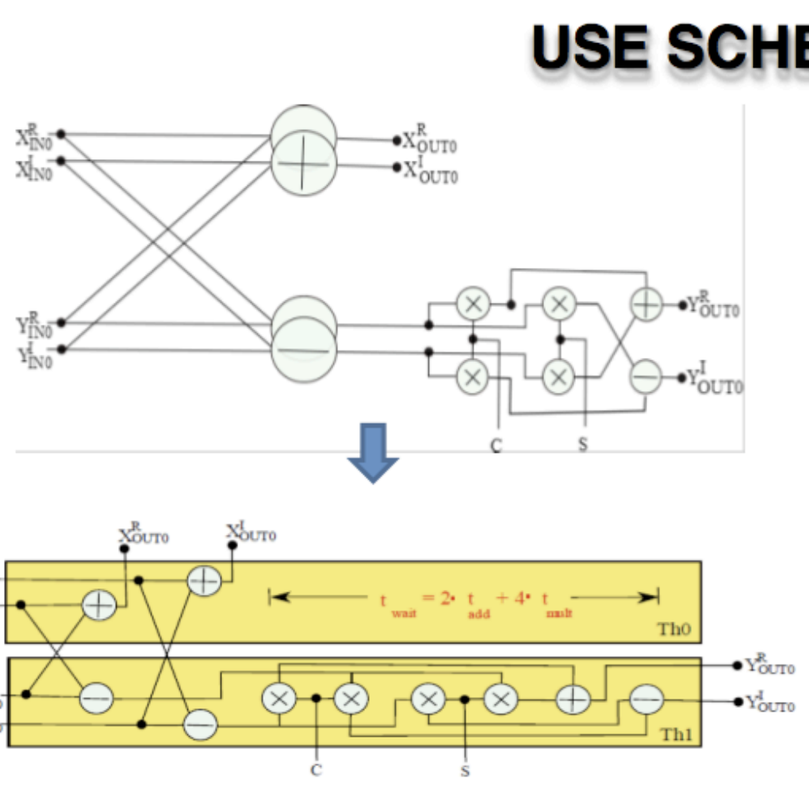


Conventional flow graph

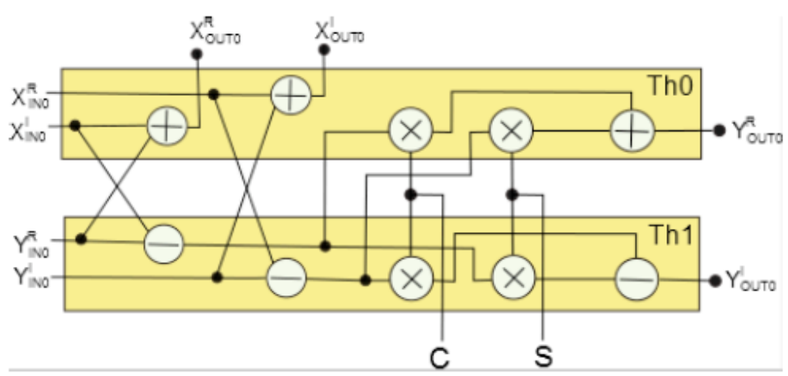


Constant Geometry

Balance load between threads



Unbalanced scheduling



Balanced scheduling

Result:

Our implementation was significantly faster that NVidia's cufft - that is, for the sizes we tried

Best paper award at the conference

Algorithms can often be modified more than it seems

Speaking of "NVidia made your bed"...:

NVidia provides a lot of extra software, libraries, tools...

cuFFT, cuBLAS, Thrust...

This can often help... but as always, make sure that somebody else's software solves *your* problem!

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