Physics for computer game developers

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About the lectures

- Purpose
  - To inroduce into basics of physics, in order to model the ”real world” in computer games

- Sources

  Webpage of the course (Ingemar R)

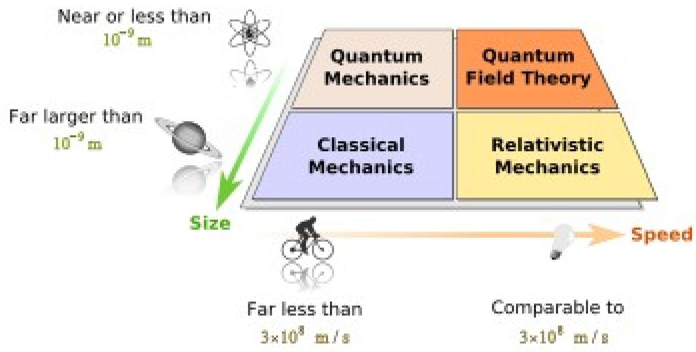
  http://computer-graphics.se/TSBK03/

  https://www.computer-graphics.se/TSBK03/files/Beachball-Physics-2023-09-05.pdf

  Ragnemalm, PFNP-SHCWMTS [IR]

  - G. Palmer, *Physics for Game Programmers*, Apress, 2005
  - Oreilly.-.Physics.for.Game.Developers.2nd.Edition.2013
  - Ian Millingston, *Game Physics Engine Development*, Elsevier, 2007
  - Witkin, Baraff, Kass, lectures from Pixar ”SIGGRAPH 2001 Course notes” [Pix], http://www.pixar.com/
  - …

i.2 Physics as a natural science



Classical Physics

Mechanics

Molecular physics

Electromagnetism

i.3 Modelling of the physical word

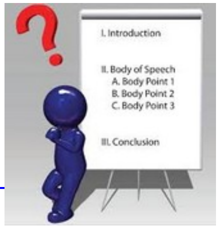
90% of games applied physical simulations use:

3D objects and 3D scenes
Movement
Rigid objects
Rotation
Friction
Air and water resistance
Gravity
Collisions and explosions
Springy things
Waves

Criterion for a physical model in a game:

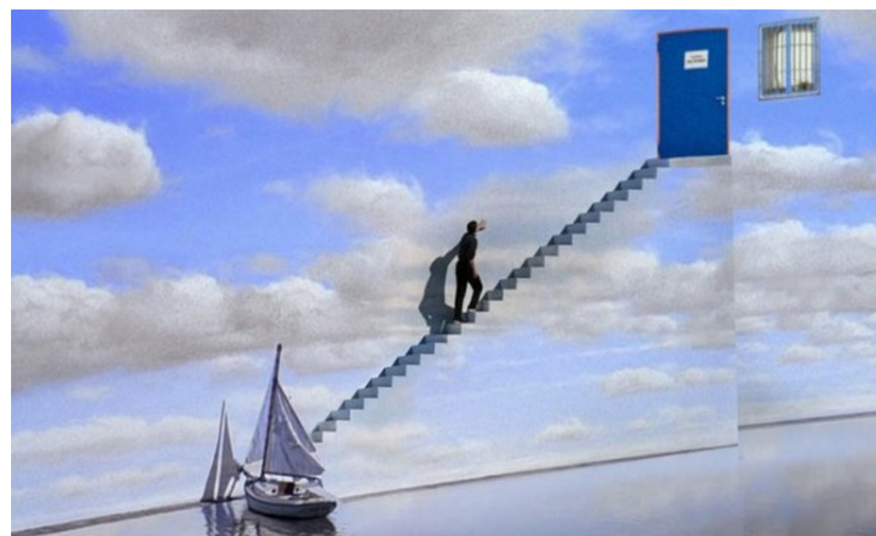
If it is looks right on the screen, that's good enough!

Outline



- *i*. Introduction (physics and its role in game industry)
- Models
- Kinematics
- Newton's Dynamics
- Work, Energy and Power
- Rotational motion
- Projectiles
- Collisions
- Water simulations
- Sports Simulations
- Cars and Motorcycles
- Boats, flight simulation (airplanes, rockets and missiles)
- Optical effects

i.3 Real word & fakes



i.3 Real word & fakes

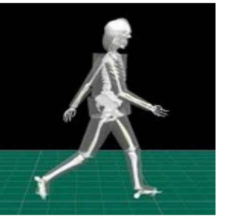
i.3 Modelling of the physical word

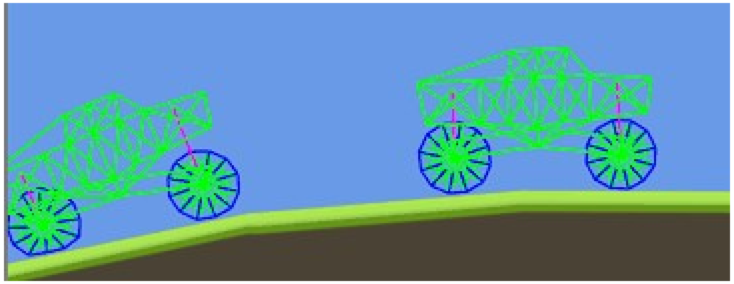
- Physics Will Keep Your Games from Looking Fake
- Adding Physics-Based Realism Is Easier Than You Might Think
- Adding Physics Won’t Affect Game Performance
- Knowing Some Physics Will Make You a Better Game Programmer

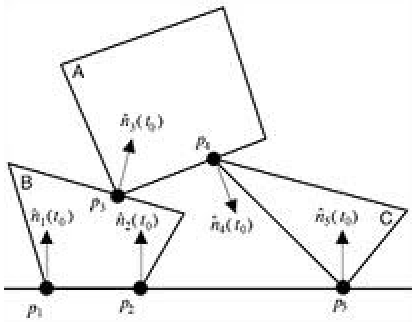
Modelling and models

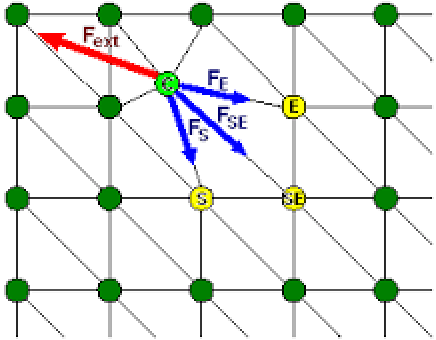
Modelling and models

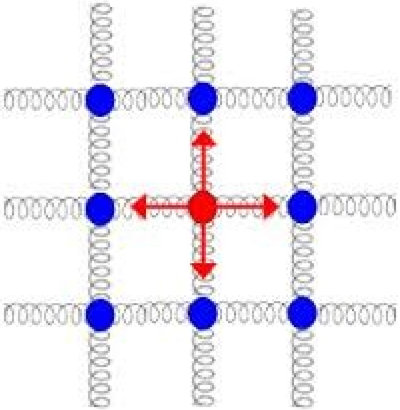
Modelling and models

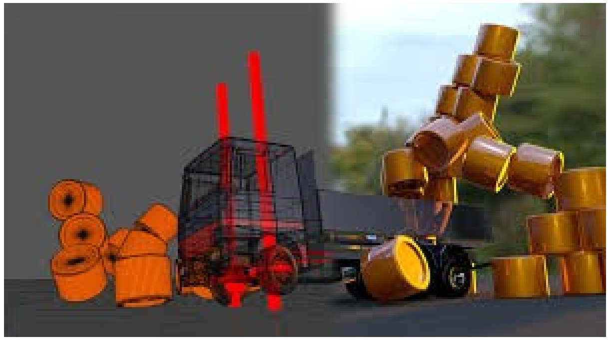




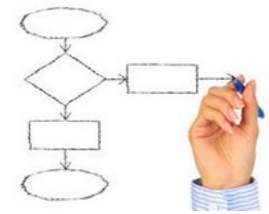




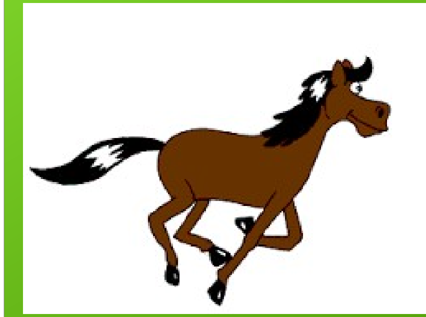




Modelling and models



Graphical unit



Coordinates of all objects

Physical unit

t=t+delta t

For (i=1; i<=Number of objects; i++)
{
Set initial conditions for the i-th object @ t;

Write Eq. of motion for the i-th object

Solving Eq. of motion for the i-th object;
}

Initial conditions for all objects @t

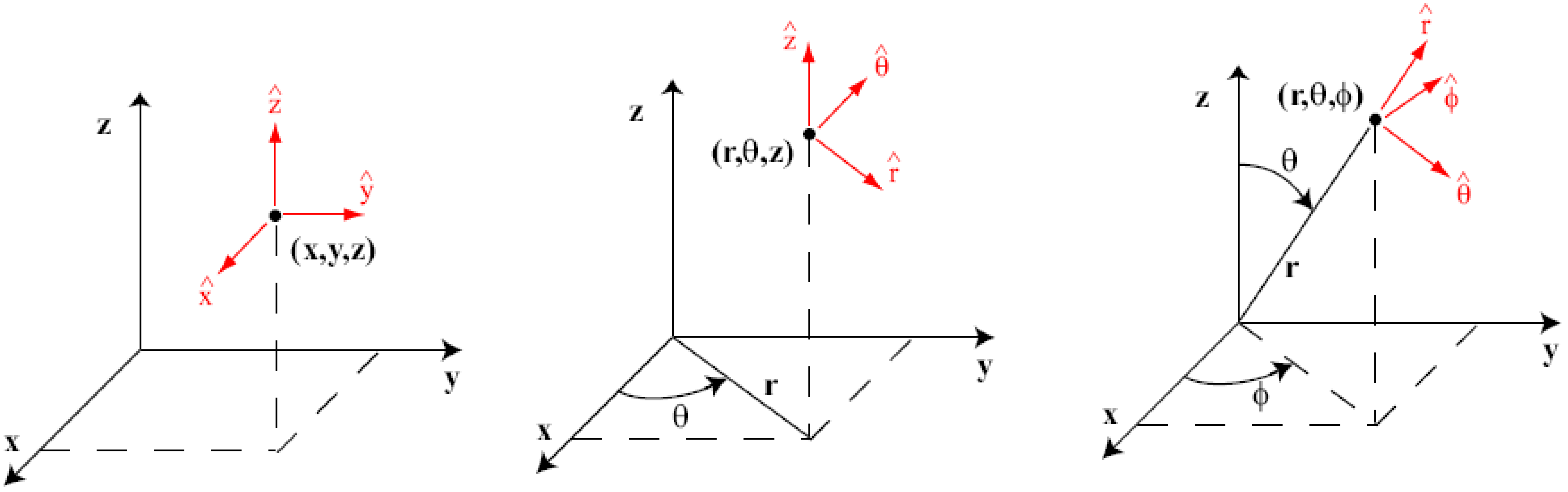
Basic Concepts

Systems of Units

Quantity	English Units	SI Units	Conversion Factor
Length	foot (*ft*)	meter (*m*)	0.3048
	mile	kilometer (*km*)	1.609
Mass	pound-mass (*lbm*)	kilogram (*kg*)	0.4536
	slug	kilogram (*kg*)	14.593
Force	pound (*lb*)	Newton (*N*)	4.448
Pressure	lb/in^2	N/m^2	6894.7
Density	$slug/ft^3$	kg/m^3	515.379
	lbm/ft^3	kg/m^3	16.018
Temperature	Fahrenheit (^oF)	Kelvin (*K*)	5/9(F + 459.67)
	Rankine (*R*)	Kelvin (*K*)	5/9

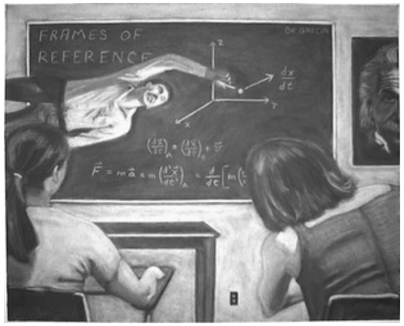
Basic Concepts

- **Coordinate Systems and Frames of Reference**

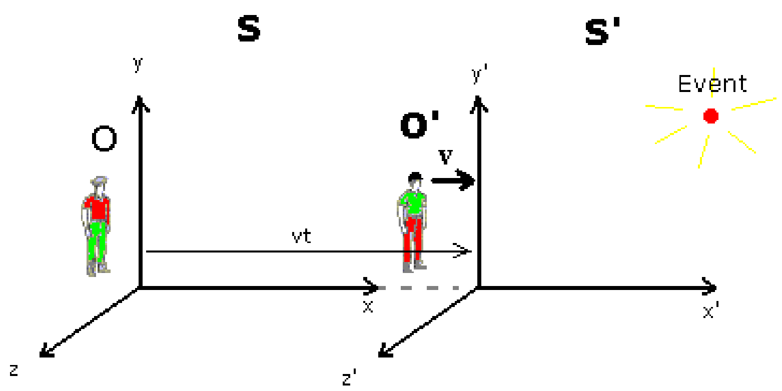


Basic Concepts

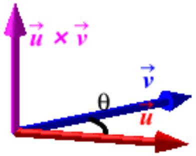
- **Coordinate Systems and Frames of Reference**



Transformation of Coordinates



The observers are moving at a relative velocity of v and each observer has their own set of coordinates (x,y,z,t) and (x',y',z',t'). What coordinates do they assign to the event?



Basic Concepts

Scalars and Vectors

$$\vec{R}=x\,\vec{i}+y\,\vec{j}+z\,\vec{k} \qquad \vec{V}=V_x\,\vec{i}+V_y\,\vec{j}+V_z\,\vec{k}$$

Magnitude of a vector: $|\vec{R}|=\sqrt{x^2+y^2+z^2}$

Sum of vectors: $\vec{R}_1\pm\vec{R}_2=(x_1\pm x_2)\,\vec{i}+(y_1\pm y_2)\,\vec{j}+(z_1\pm z_2)\,\vec{k}$

Vector scalar product: $(\vec{R}_1\cdot\vec{R}_2)=x_1x_2+y_1y_2+z_1z_2$

$$(\vec{R}_1\cdot\vec{R}_2)=|\vec{R}_1||\vec{R}_2|\cos\alpha$$

Vector cross product: $[\vec{R}_1\times\vec{R}_2]=(y_1z_2-y_2z_1)\,\vec{i}+(z_1x_2-z_2x_1)\,\vec{j}+(x_1y_2-x_2y_1)\,\vec{k}$

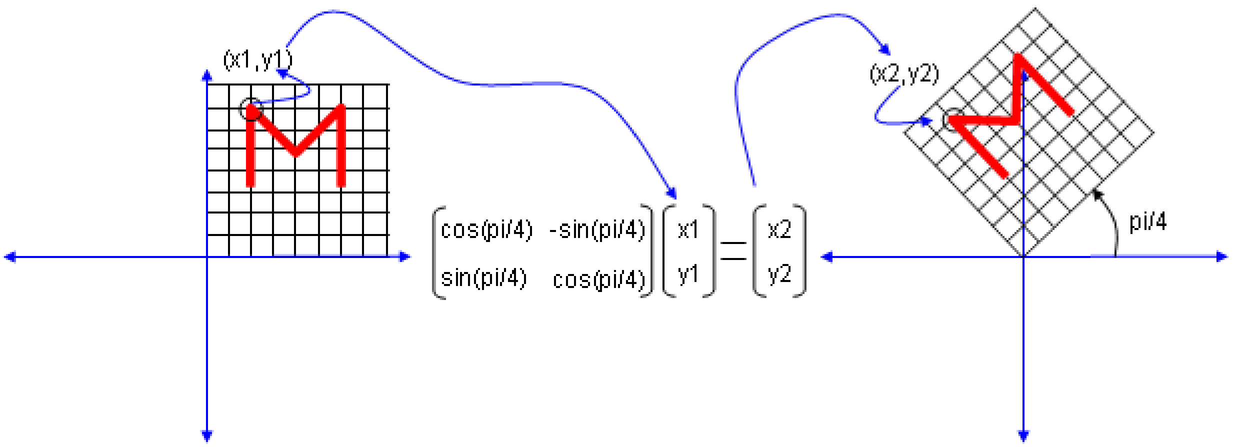
$$|[\vec{R}_1\times\vec{R}_2]|=|\vec{R}_1||\vec{R}_2|\sin\alpha$$

Basic Concepts

- **Matrices**
- **Derivatives**
- **Differential Equations**

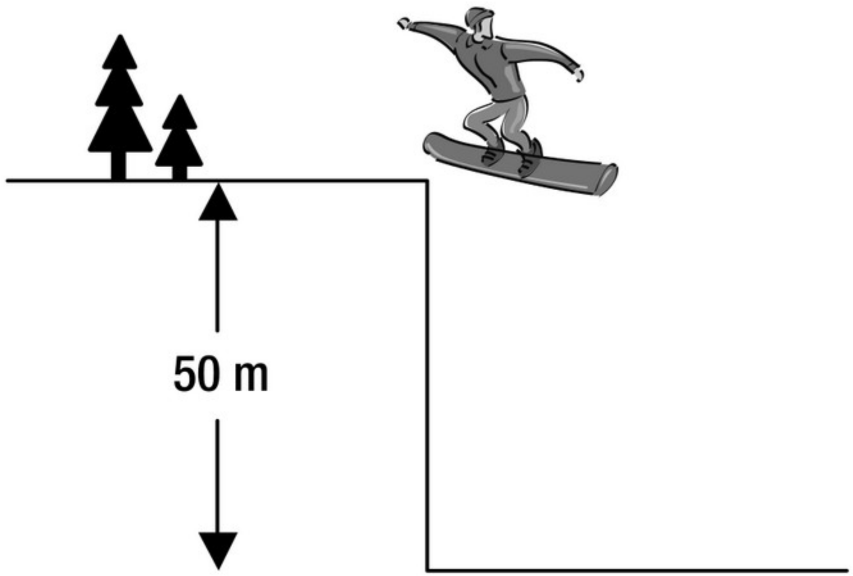
Basic Concepts

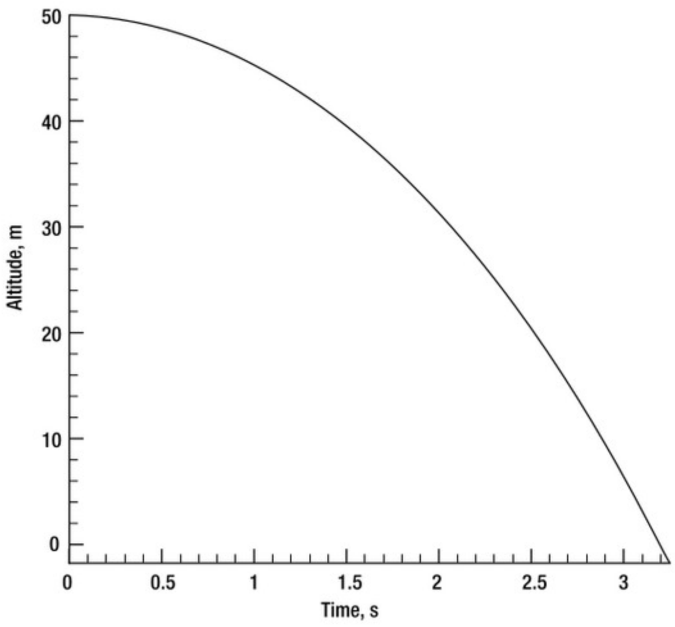
- **Matrices**



Basic Concepts

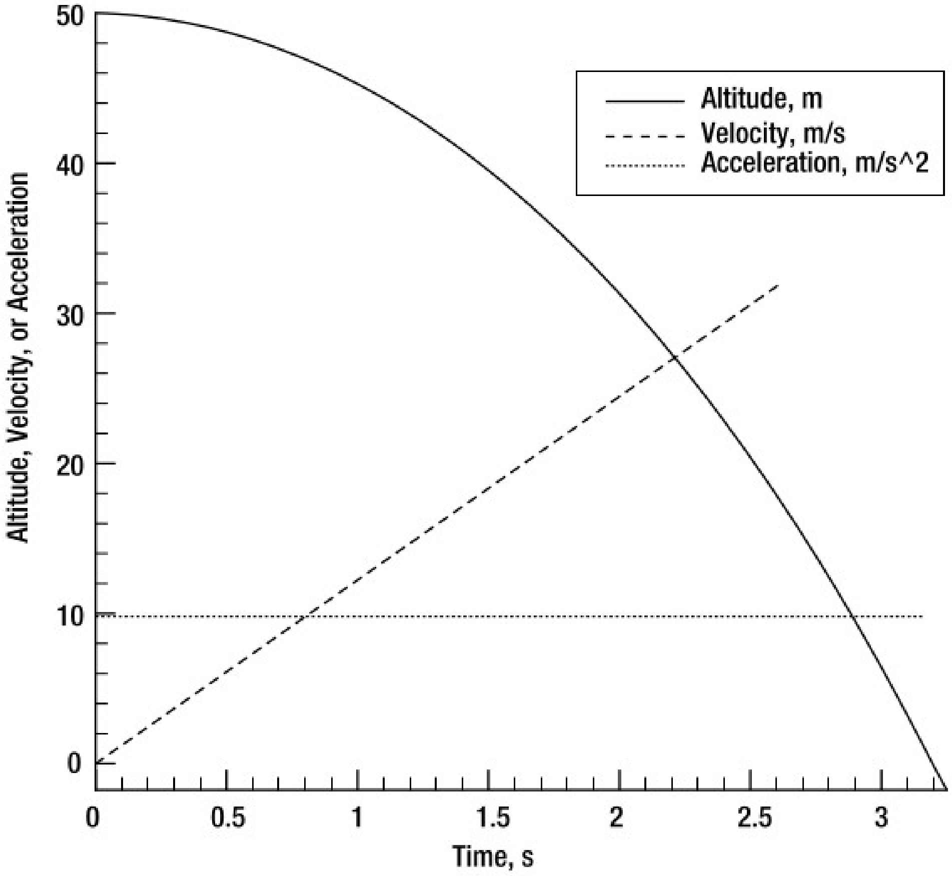
- **Derivatives**





Basic Concepts

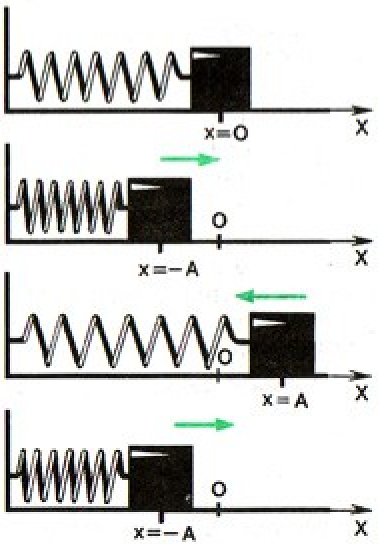
- **Derivatives**



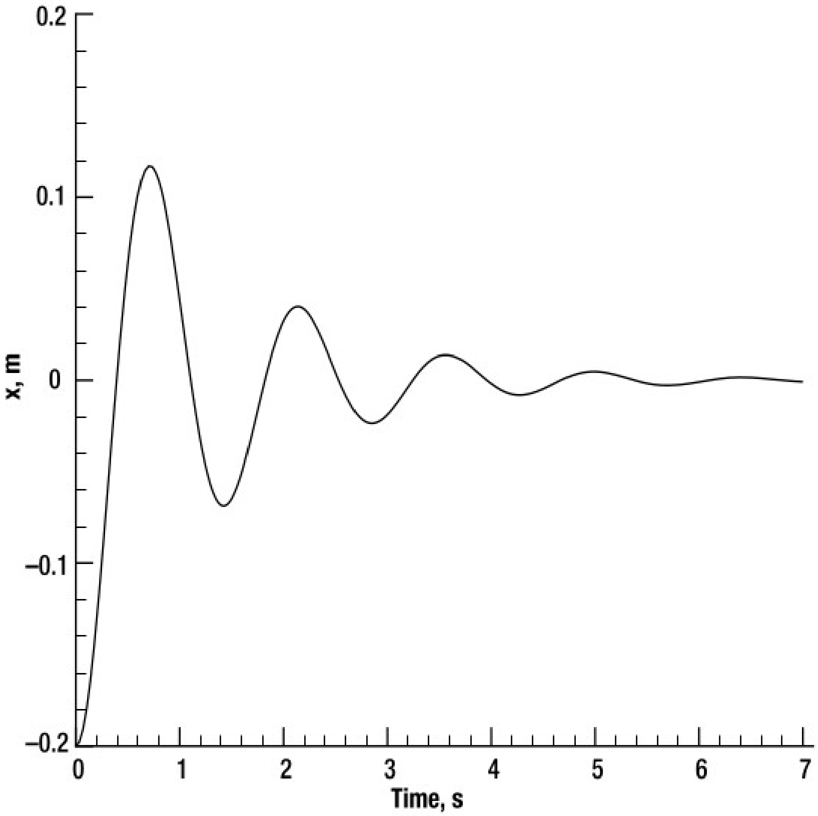
Comparing acceleration, velocity, and altitude for the snowboarder

Basic Concepts

- **Differential Equations**

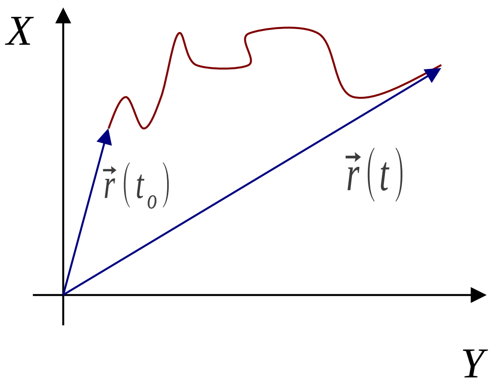


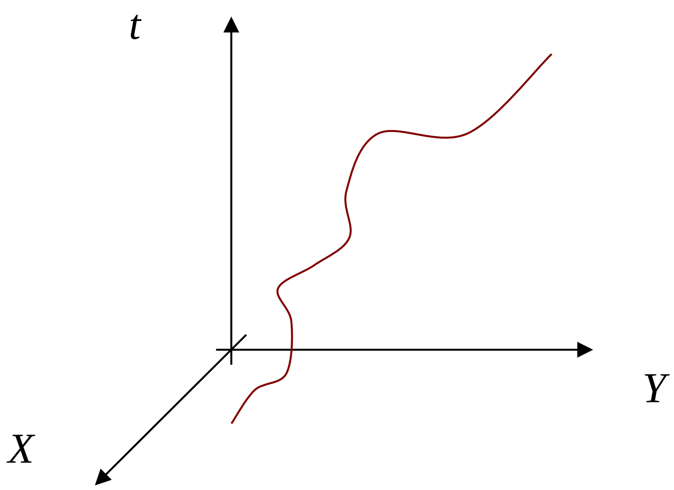
$$m\frac{d^2x}{dt^2}+\mu\frac{dx}{dt}+kx=0$$



The motion of a spring as a function of time

Kinematics





Displacement $\Delta\vec{r}(t)=\vec{r}(t)-\vec{r}(t_o)$ $[m]$

Average velocity $\vec{v}_{average}=\dfrac{\vec{r}(t)-\vec{r}(t_o)}{t-t_o}$ $\left[\dfrac{m}{s}\right]$

Instantaneous velocity $\vec{v}(t)=\dfrac{d\vec{r}(t)}{dt}=\dot{\vec{r}}(t)$ $\left[\dfrac{m}{s}\right]$

Acceleration $\vec{a}(t)=\dfrac{d\vec{v}(t)}{dt}=\ddot{\vec{r}}(t)$ $\left[\dfrac{m}{s^2}\right]$

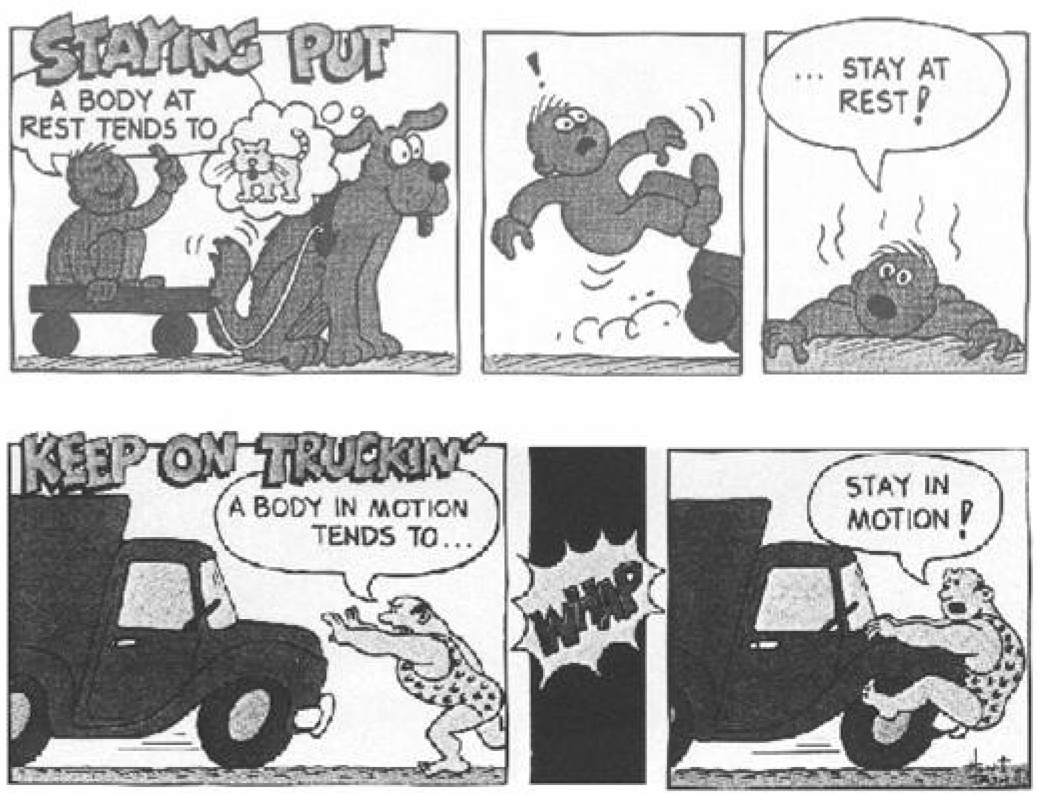
Distance $s=\int_{t_1}^{t_2}|dr|$ $[m]$

$$\vec{r}(t)=\vec{r}(t_o)+\int_{t_o}^{t}\vec{v}(t)dt=\vec{r}(t_o)+\vec{v}(t_o)t+\int_{t_o}^{t}\int_{t_o}^{t}\vec{a}(t)dt$$

Newtonian Dynamics

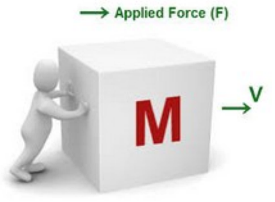
- Newton’s three laws of motion
- Some special types of forces—gravitational, friction, centripetal, and spring
- The concept of a force vector
- Force balances and force diagrams

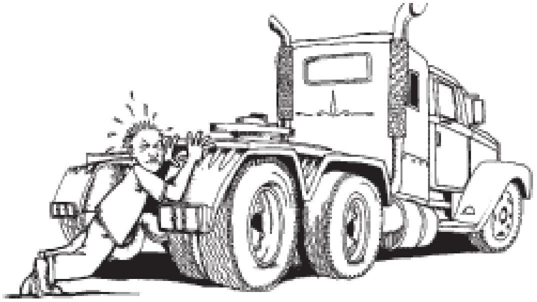
Newton's First Law of Motion: Inertia

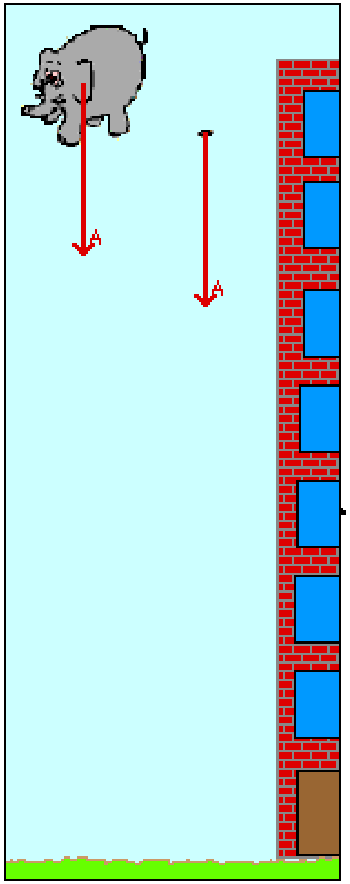


Every body preserves in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.

Newton’s Second Law of Motion: Force, Mass, and Acceleration



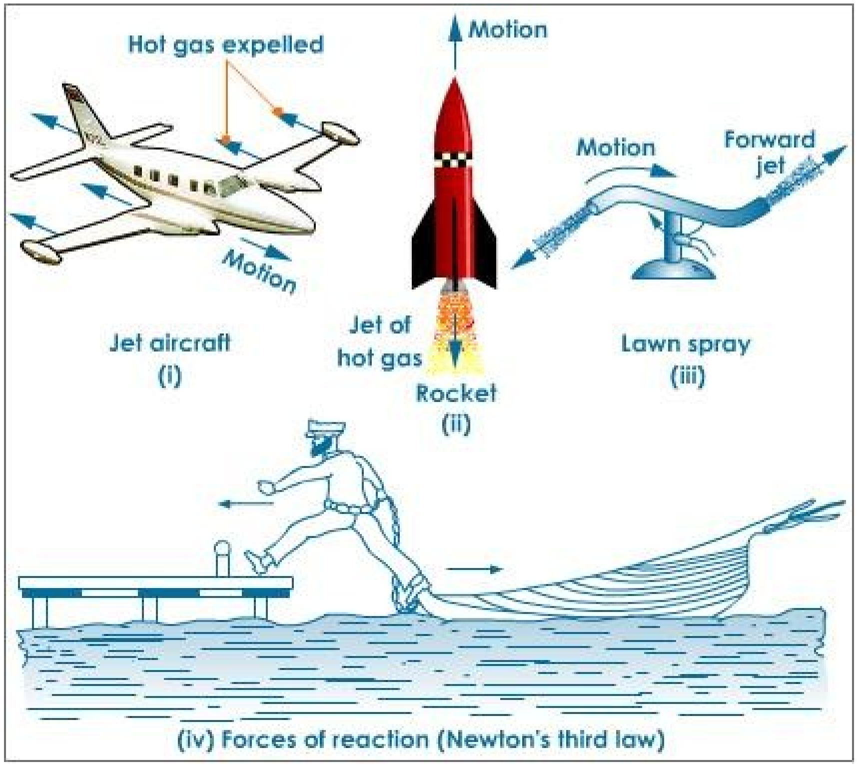




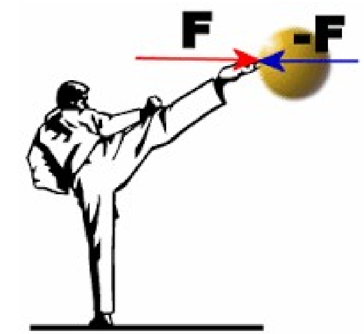
$$\vec{F} = m\vec{a}$$

The alteration of motion is ever proportional to the motive force impressed

Newton’s Third Law of Motion: Equal and Opposite Forces



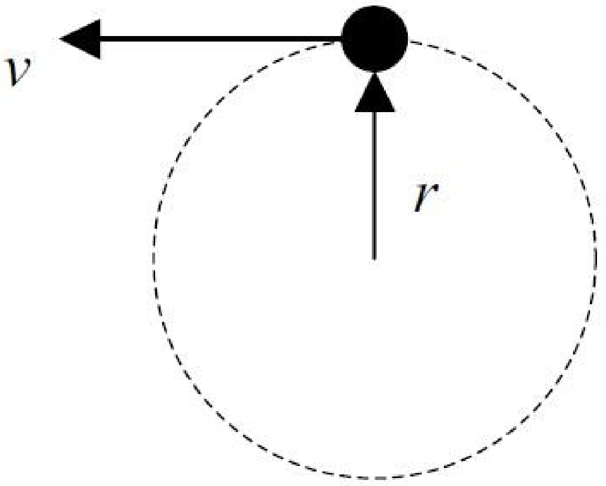
(iv) Forces of reaction (Newton's third law)

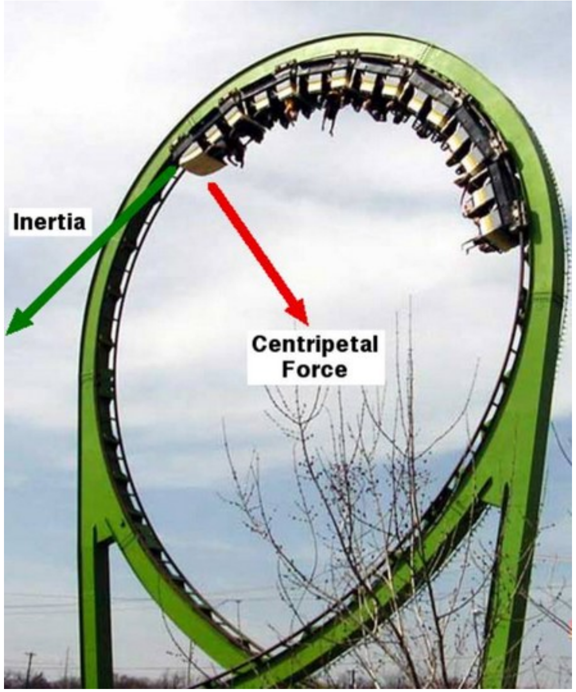


To every action there is always opposed an equal reaction

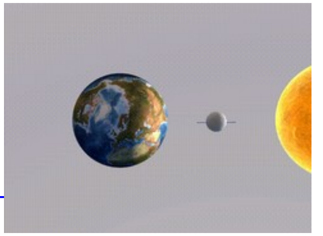
Types of Forces

- **Centripetal Force**



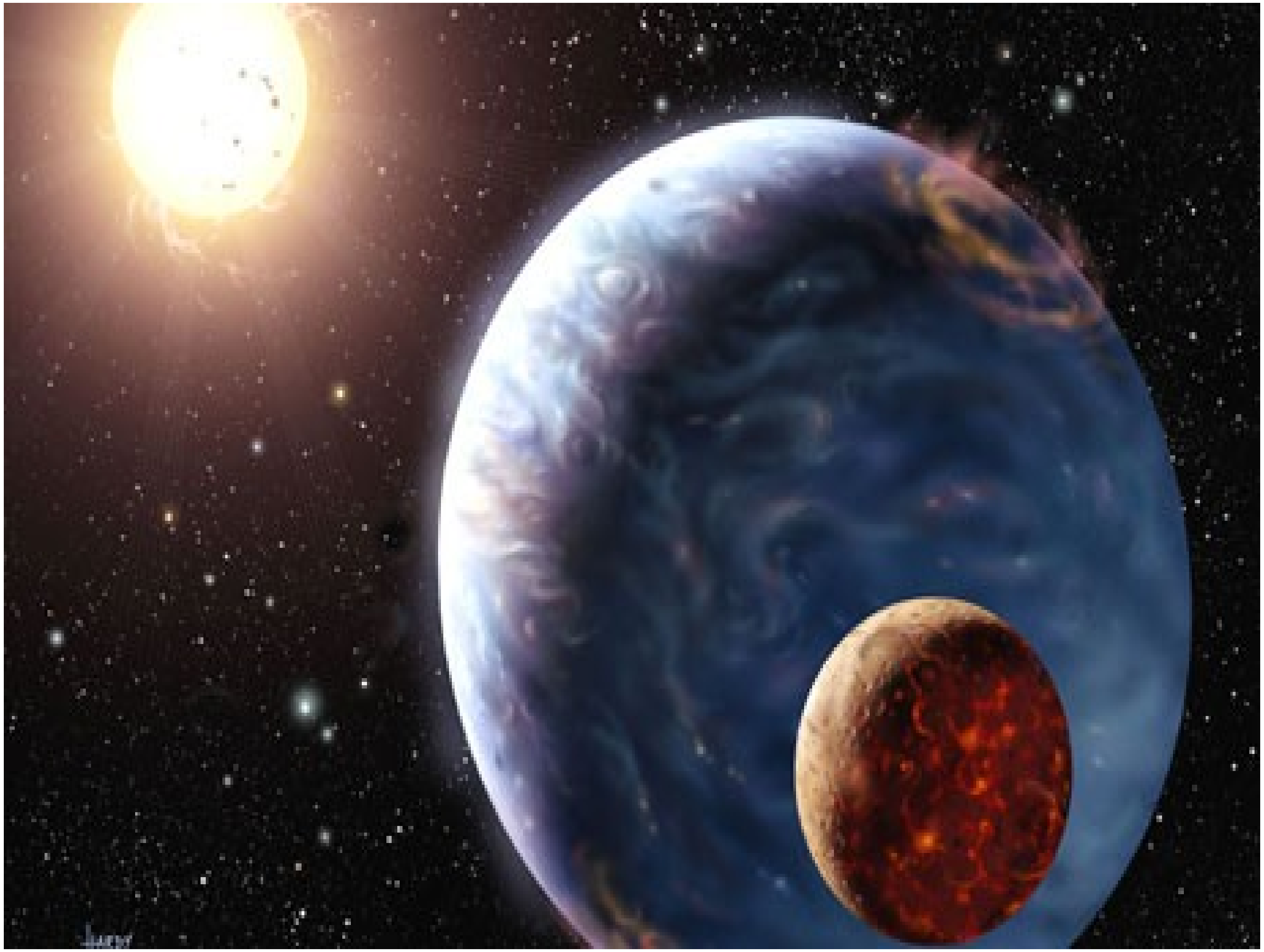


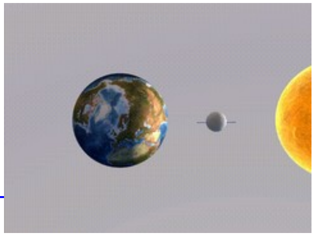
$$F = \frac{mv^2}{r}$$



Types of Forces

- **Gravitational Force**





Types of Forces

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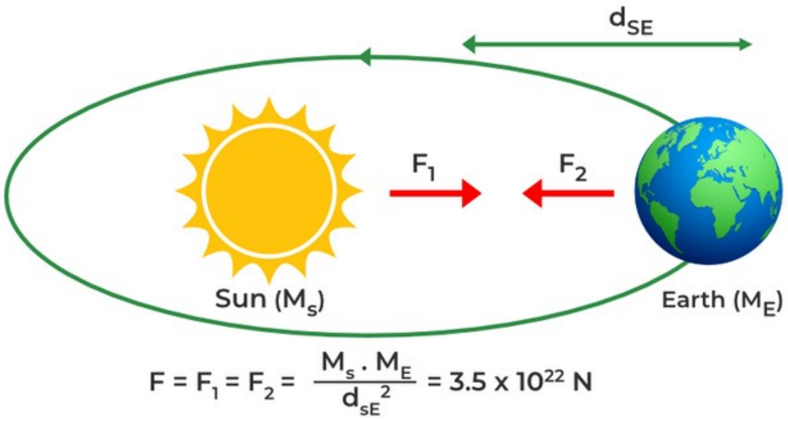
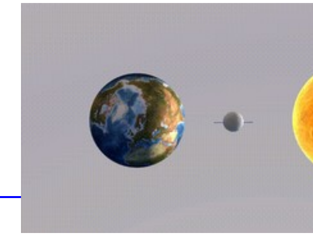


$$\vec{F} = G \frac{M_1 M_2}{|\vec{R}_{12}|^3} \vec{R}_{12} \quad = M_2 g$$

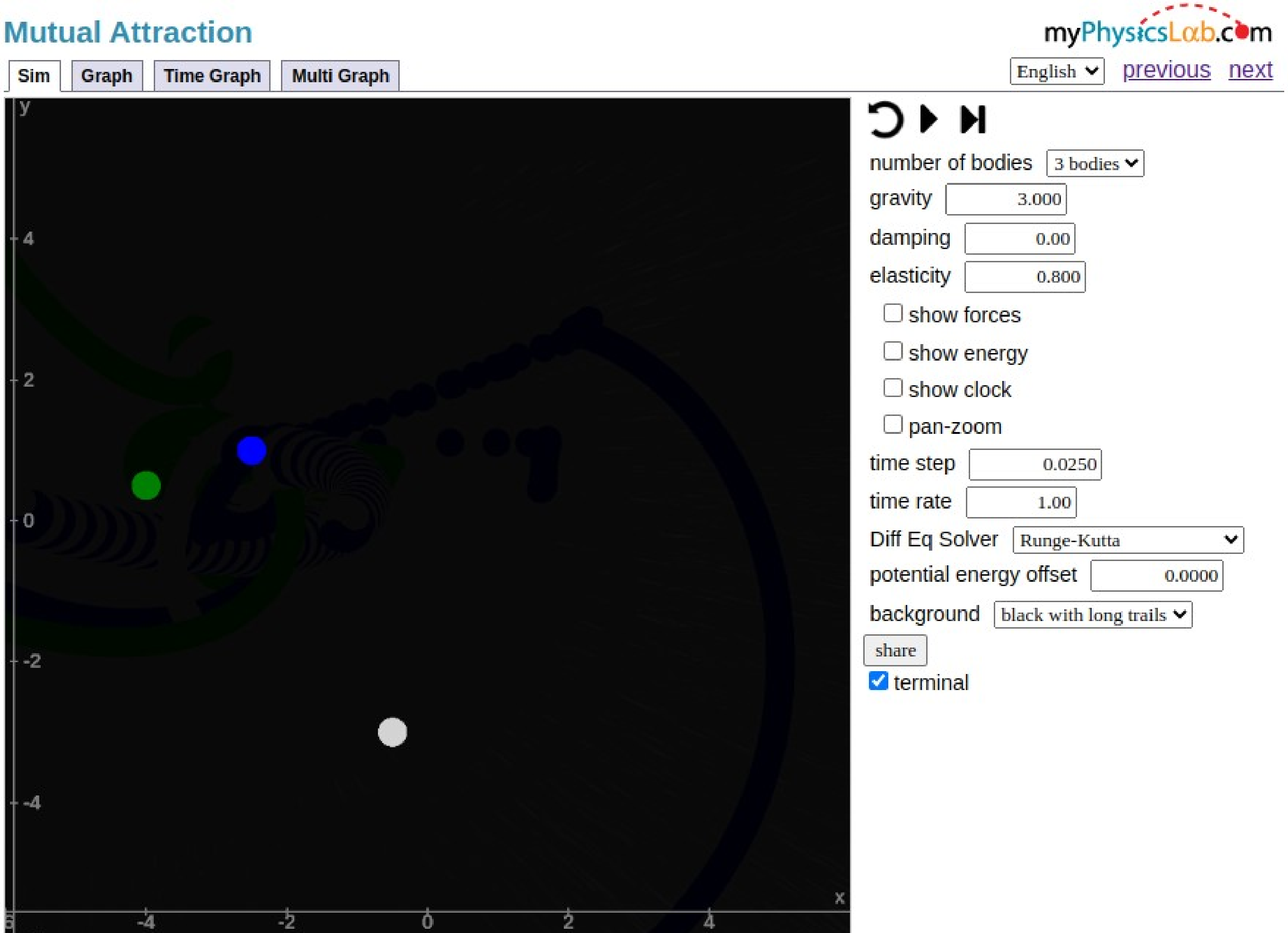
$$G = 6.674 \cdot 10^{-11} \frac{Nm^2}{kg^2}$$

$$g = 6.674 \cdot 10^{-11} \frac{Nm^2}{kg^2} \frac{5.9736 \cdot 10^{24} kg}{(6,375 \cdot 10^6 m)^2} = 9.81 \frac{N}{kg}$$



Types of Forces

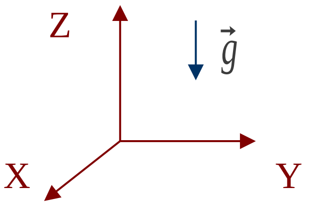
Gravitational Force



https://www.myphysicslab.com/engine2D/mutual-attract-en.html

Gravitation Force

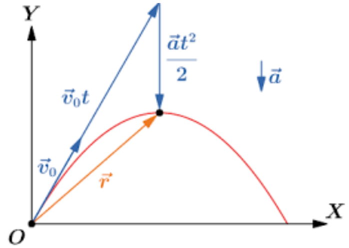
- **Equations of motion for projections**



Quantity	Differential Equation	Solution
Acceleration	None	$a_z = -g$, $a_x = 0, \quad a_y = 0$
Velocity	$\frac{dv_z}{dt} = a_z = -g$	$v_z = v_{z0} - gt$, $v_x = v_{x0}, \quad v_y = v_{y0}$
Location	$\frac{d^2 z}{dt^2} = a_z = -g$ $\frac{dz}{dt} = v_z = v_{z0} - gt$	$z = z_o + v_{z0} t - \frac{1}{2} g t^2$, $x = x_o + v_{x0} t$, $y = y_o + v_{y0} t$

Gravitation Force

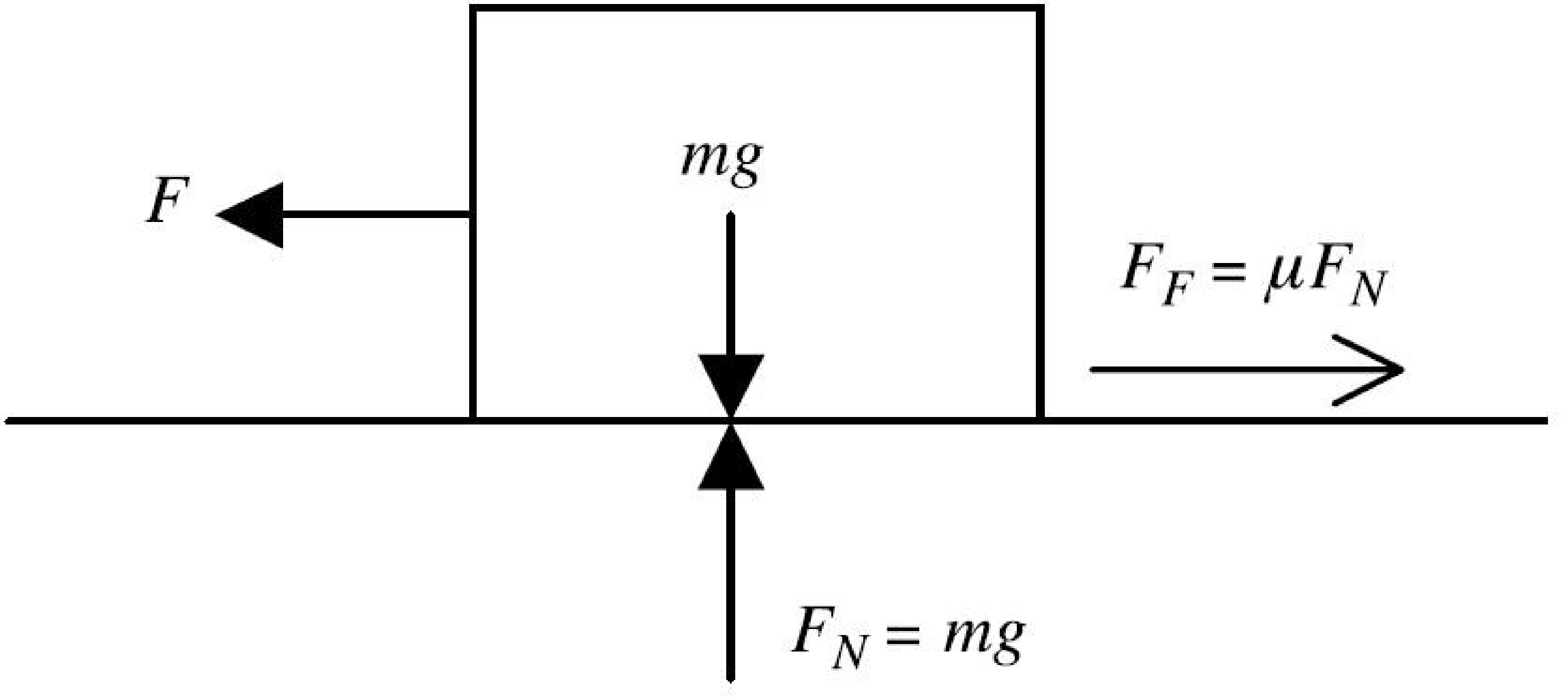
- **Equations of motion in vector form**

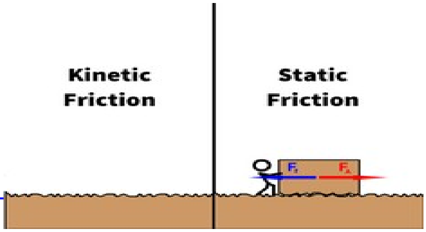


Quantity	Differential Equation	Solution
Acceleration	None	$\vec{a} = \vec{g}$
Velocity	$\frac{d\vec{v}}{dt} = \vec{a} = \vec{g}$	$\vec{v} = \vec{v}_o + \vec{g}t$
Location	$\frac{d^2\vec{r}}{dt^2} = \vec{a} = \vec{g}$ $\frac{d\vec{r}}{dt} = \vec{v} = \vec{v}_o + \vec{g}t$	$\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{g}t^2$

Friction

$$F_F = \mu F_N$$





Friction

- *Friction Coefficients for Some Common Surface Interactions*

Materials	μ_s	μ_k
Steel—steel	0.7–0.74	0.57–0.6
Steel—steel (lubricated)	0.12	0.07
Aluminum—steel	0.61	0.47
Copper—steel	0.53	0.36
Cast iron—cast iron	1.1	0.15
Teflon—Teflon	0.04	0.04
Glass—glass	0.94	0.4
Wood—wood	0.25–0.5	0.2–0.3
Rubber—concrete	1.0	0.8
Rubber—concrete (wet)	0.7	0.5
Ice—ice	0.1	0.03
Waxed ski—snow	0.1–0.14	0.05–0.1

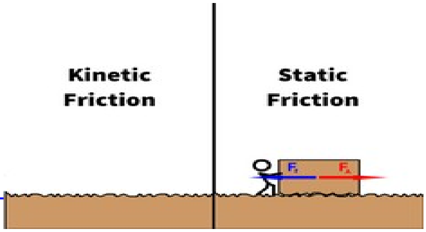
* *Source: RoyMech,* `www.roymech.co.uk`

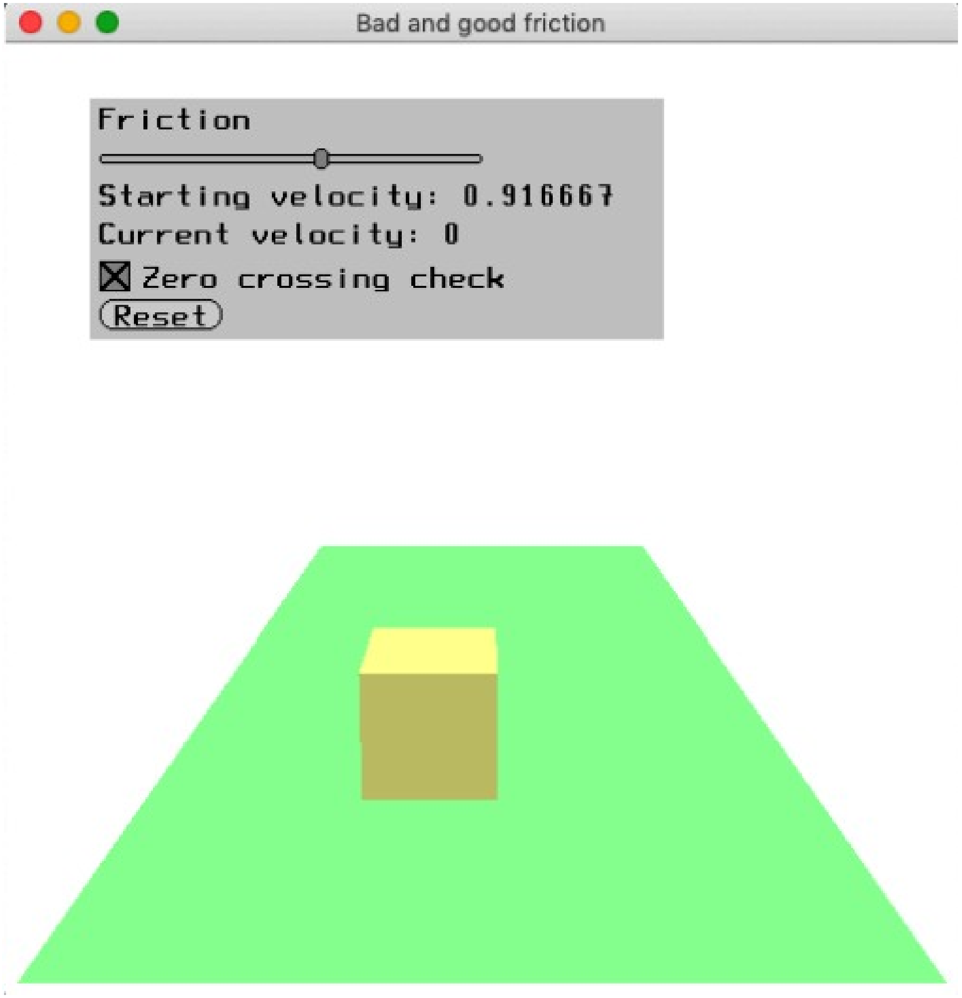
* *Raymond Serway and John Jewitt,* Physics for Scientists and Engineers, Sixth Edition *(Brooks-Cole, 2003)*

* `www.physlink.com/Reference/FrictionCoefficients.cfm`

* *Encarta.msn.com*

Friction

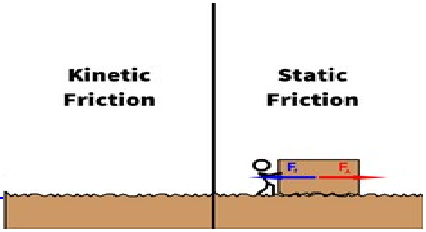


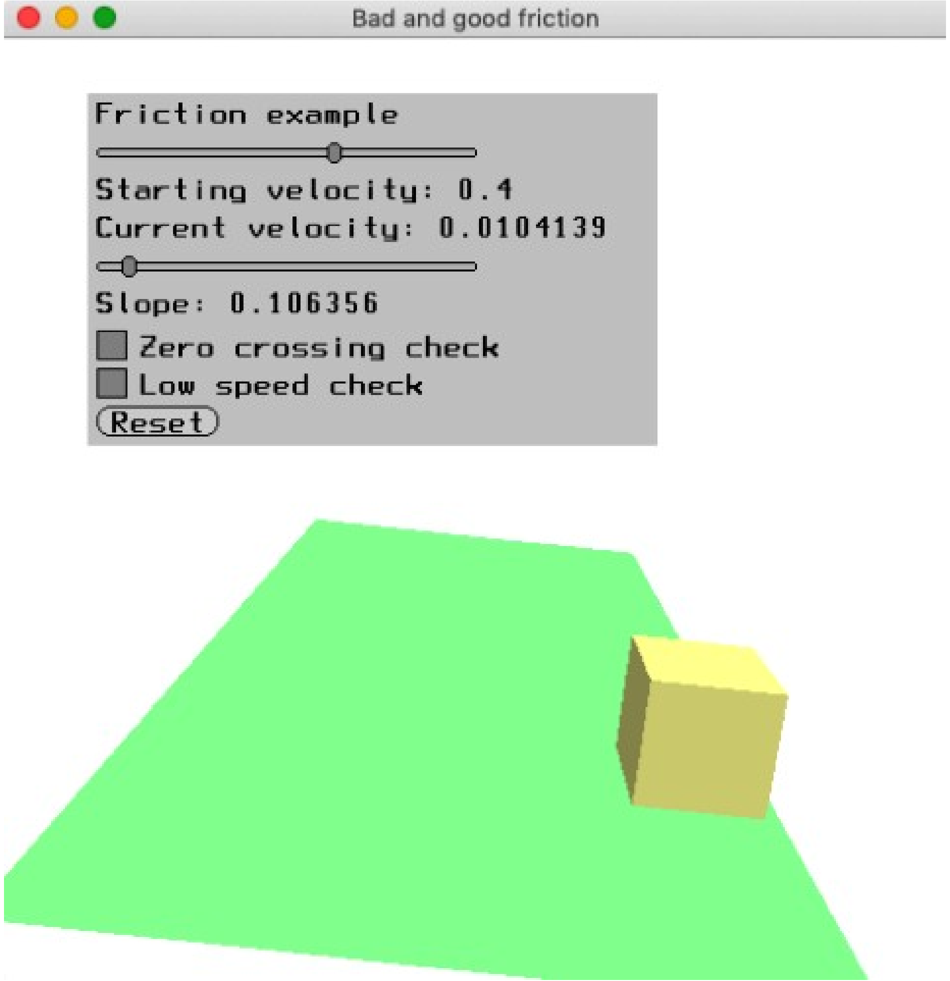


The sliding block demo

https://www.computer-graphics.se/demos/files03/Beachball-demos.zip

Friction





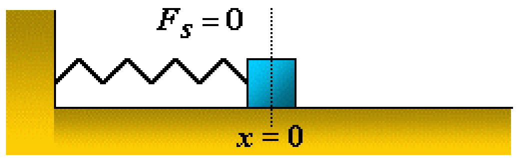
The sliding block demo 2

https://www.computer-graphics.se/demos/files03/Beachball-demos.zip

Vibration

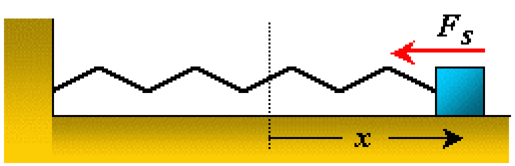
- **Deformation**

Springs



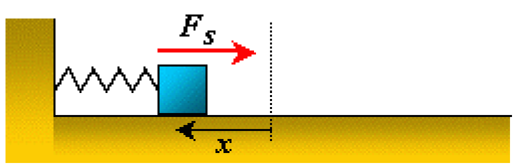
Hooke's Law

$$\vec{F} = -k\Delta\vec{x}$$



Equation of motion: $m\ddot{x} = -kx$

$$\ddot{x} + \varpi^2 x = 0 \qquad \varpi^2 = \frac{k}{m}$$



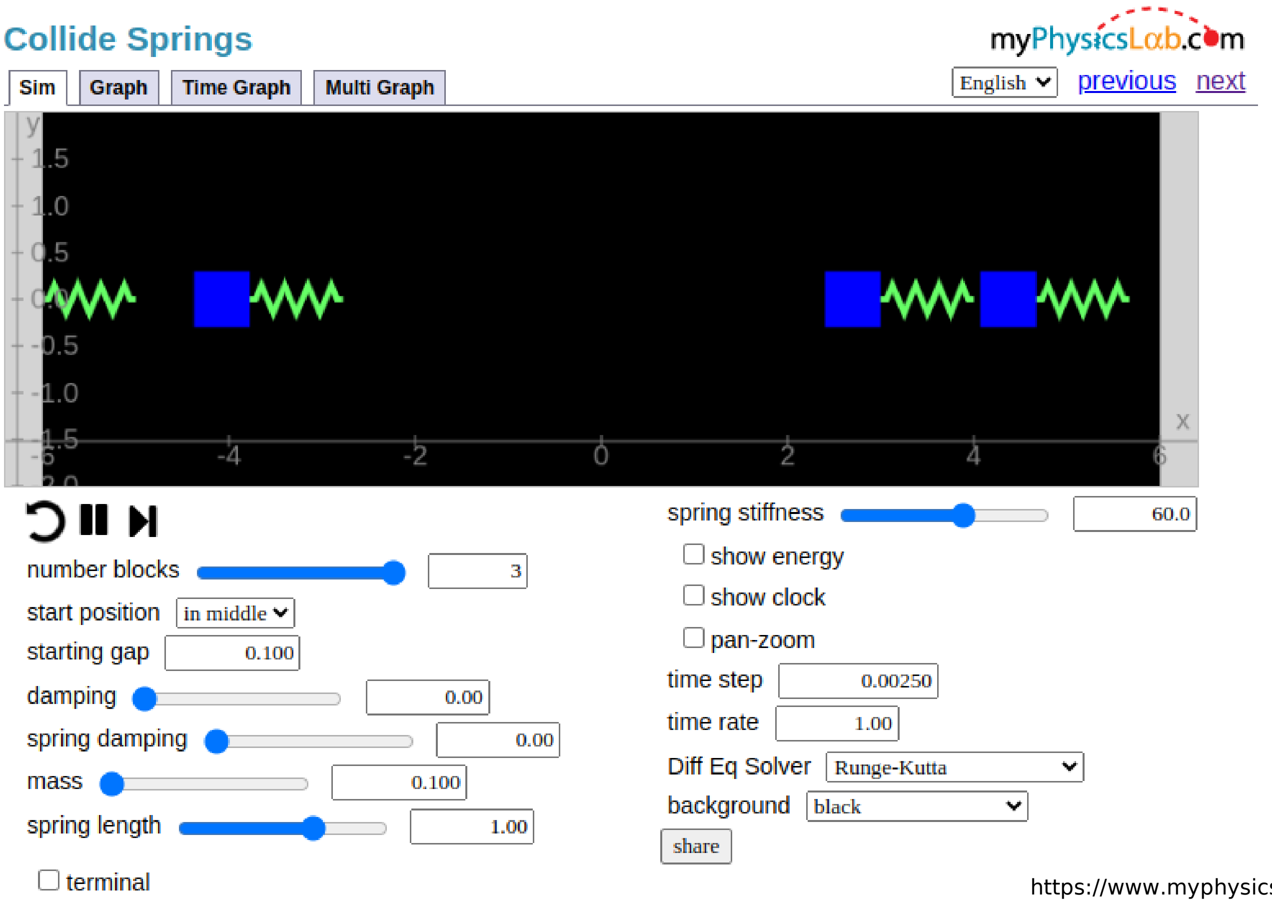
Solution: $x(t) = A\sin(\varpi t + \phi_o)$

$$T = \frac{2\pi}{\varpi} = 2\pi\sqrt{\frac{m}{k}}$$

Vibration

- ## Deformation

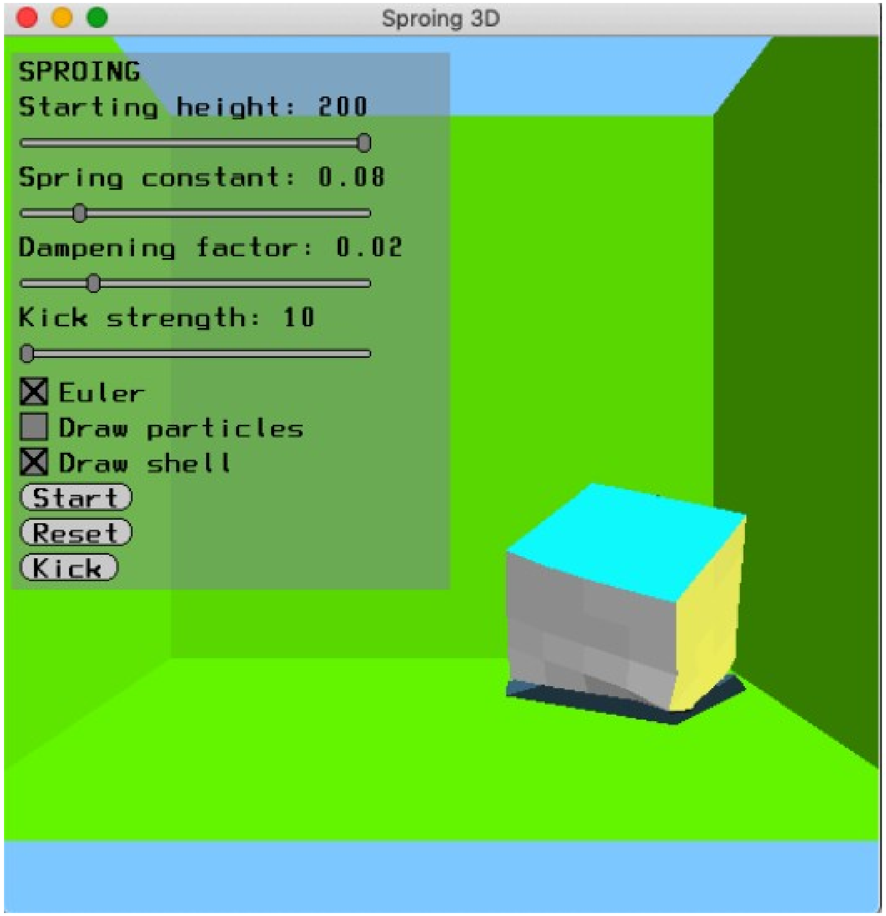
  ### Springs



https://www.myphysicslab.com/springs/collide-spring-en.html

Vibration

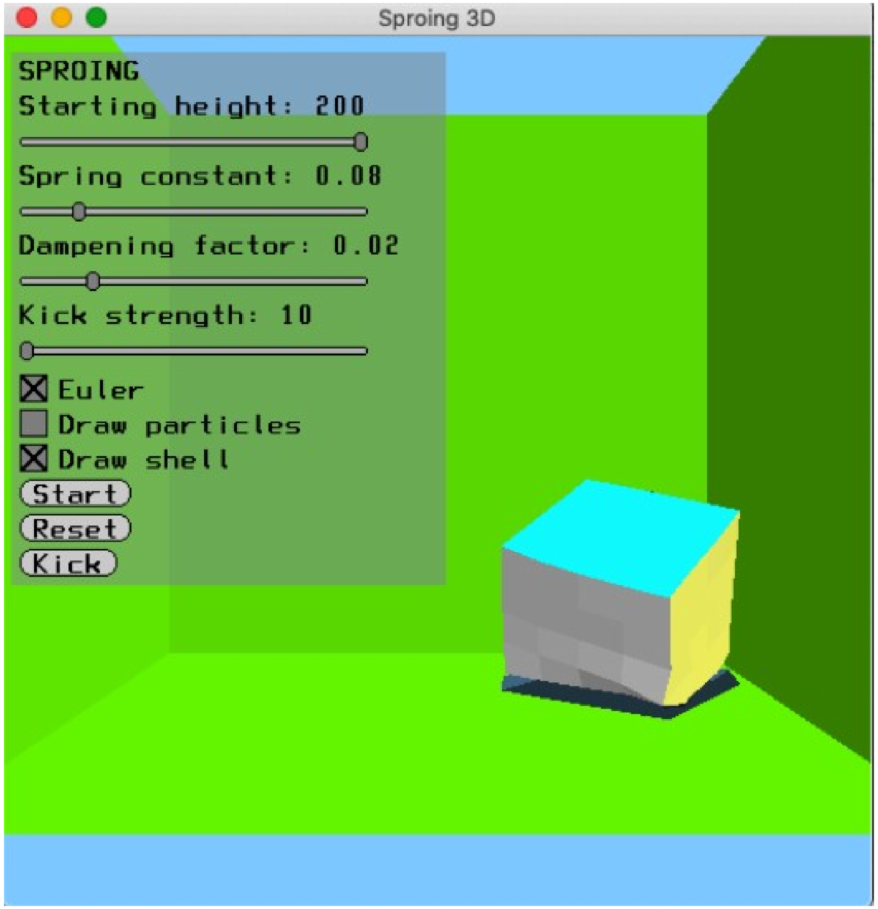
Deformation

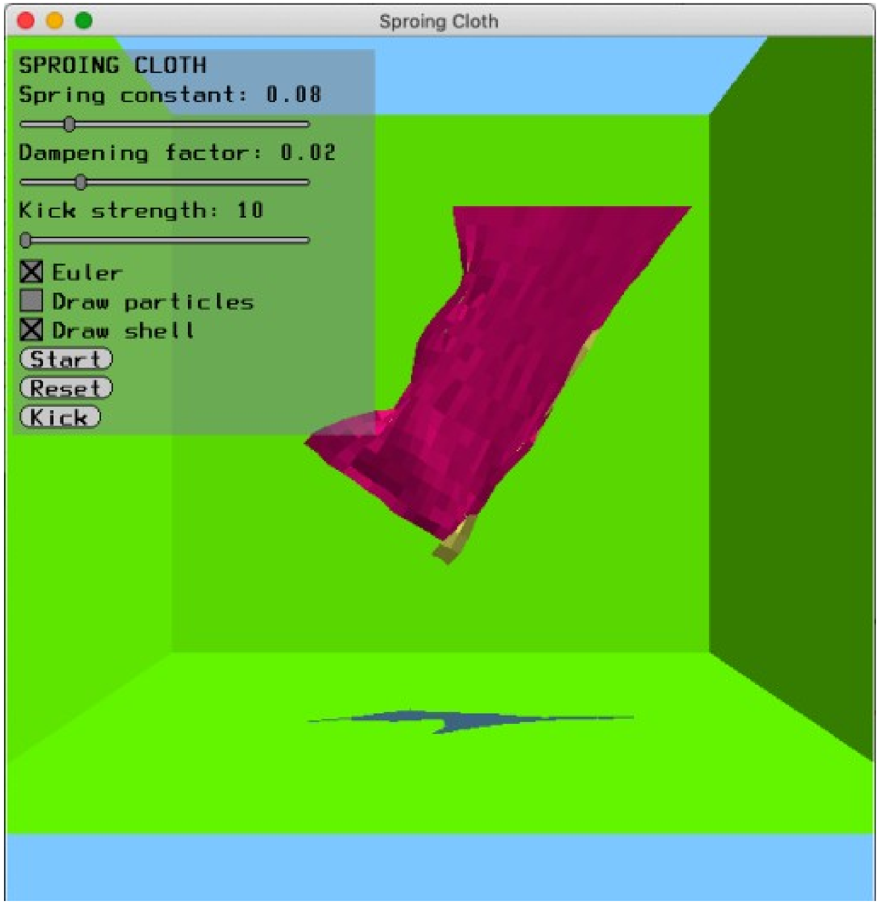


https://www.computer-graphics.se/demos/files03/Beachball-demos.zip

Vibration

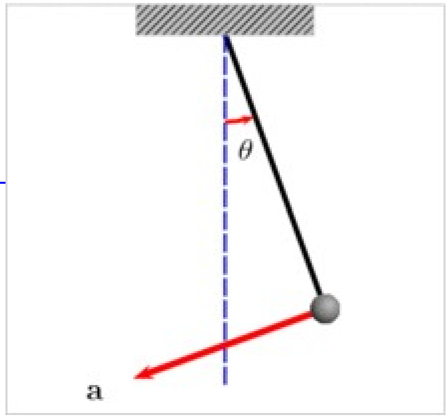
■ **Deformation**





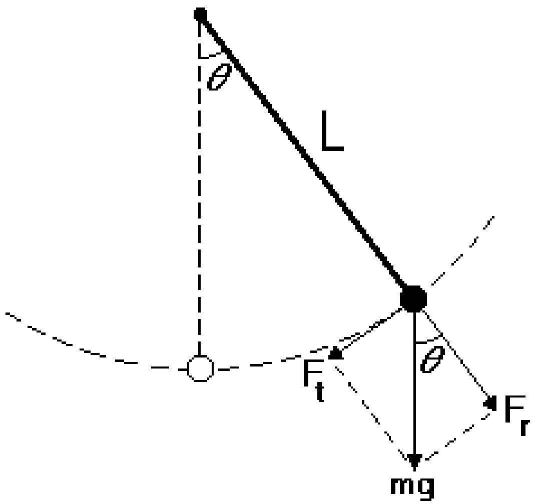
https://www.computer-graphics.se/demos/files03/Beachball-demos.zip

Vibration



- **Pendulum**

$$\vec{F} = -mg\sin\theta = -\frac{mg}{L}\Delta\vec{x}$$



$$\vec{F} = -k\Delta\vec{x}$$

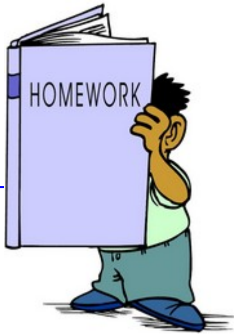
Equation of motion: $m\ddot{x} = -kx$

$$\ddot{x} + \varpi^2 x = 0 \qquad \varpi^2 = \frac{g}{L}$$

Solution: $x(t) = A\sin(\varpi t + \phi_o)$

$$T = \frac{2\pi}{\varpi} = 2\pi\sqrt{\frac{L}{g}}$$

Vibration



- **Boat**

Write the equation of motion for a boat in water.

What is the period of the vibration?