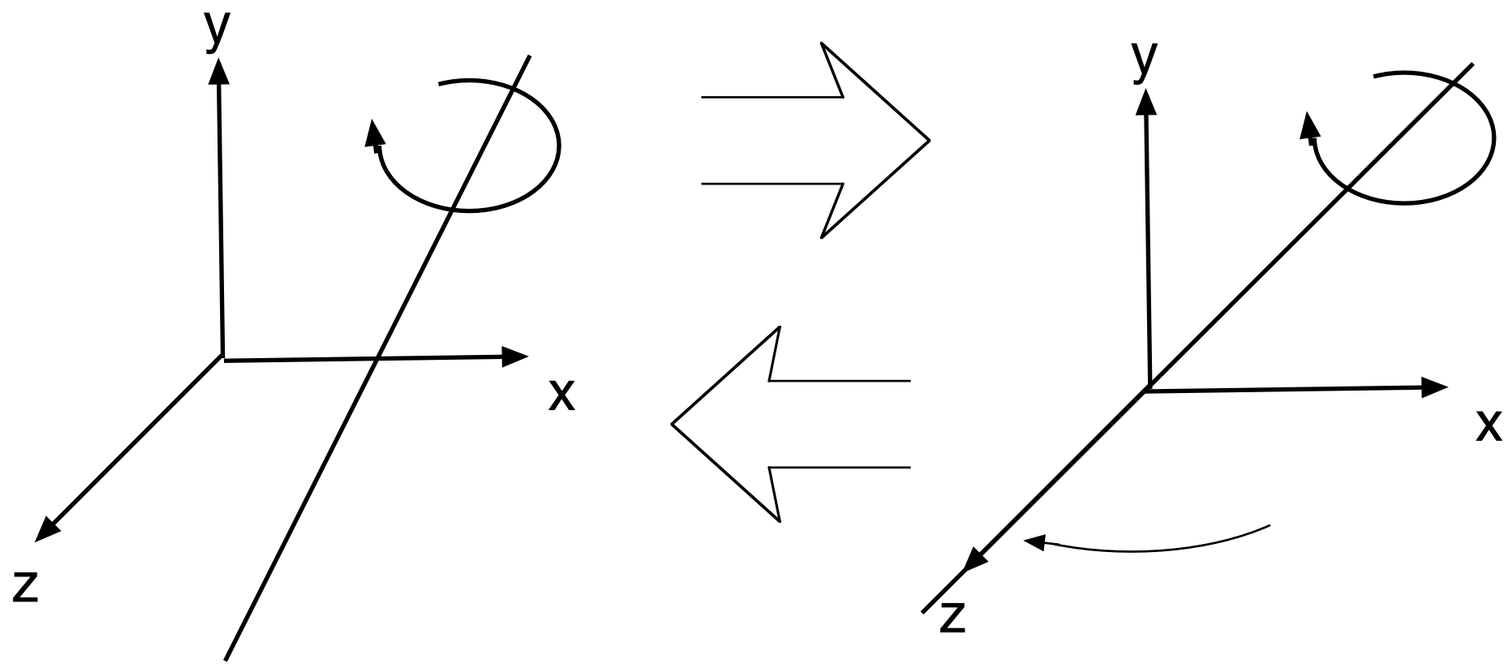
More transformations

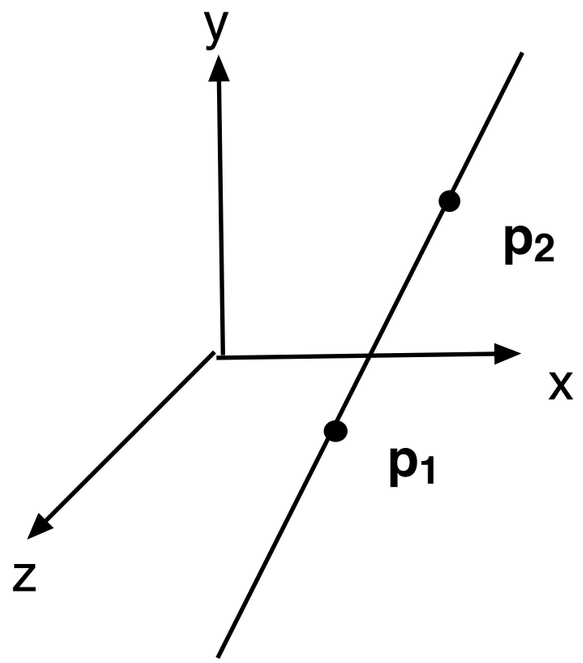
Rotation around arbitrary axis

Rotation around arbitrary axis



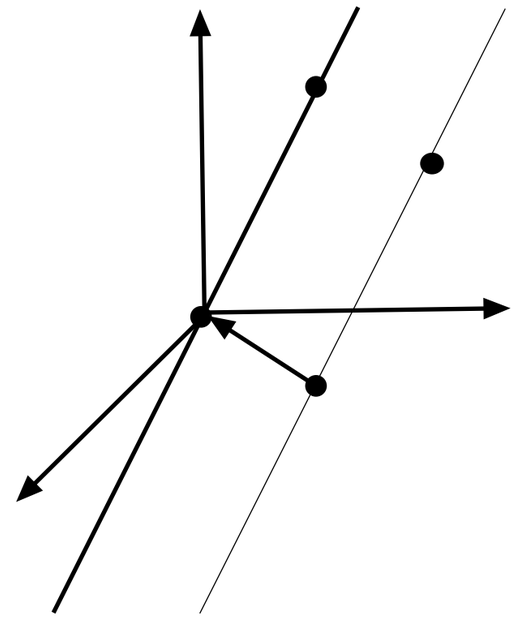
Transform to align axis with the Z axis, rotate, and transform back.

Definition of the rotation axis



p_1 and p_2 define the rotation axis

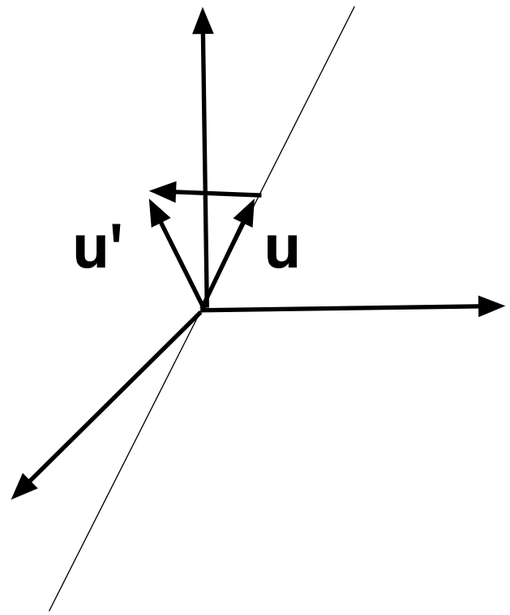
Translate to origin



$T(-p_1)$

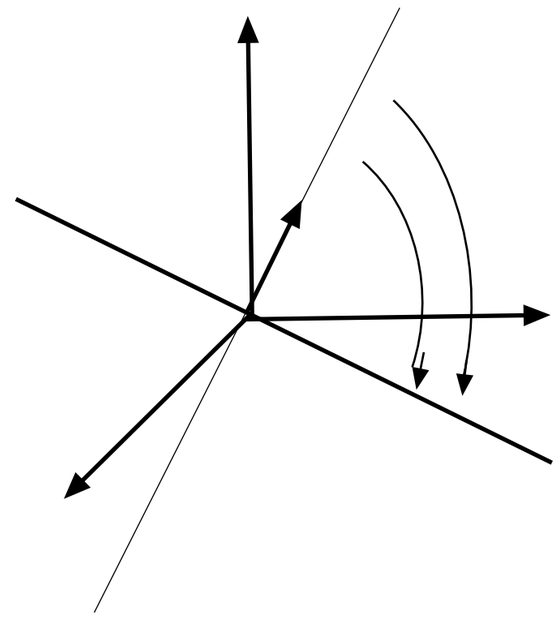
Geometrical method

Finding an angle to rotate around X



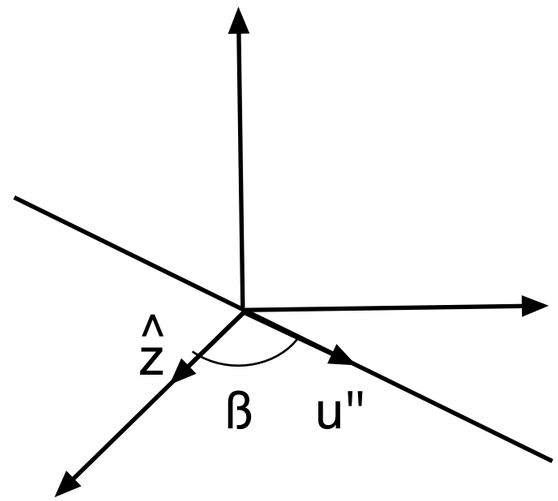
Project u on the yz plane = (0, u_y, u_z)

Rotate around X



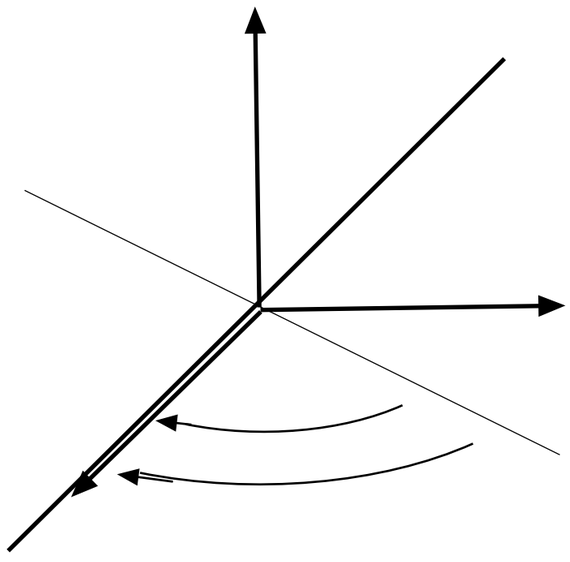
$R_x(\alpha)$

Finding an angle to rotate around Y



u'' and $\hat{z}$ gives the angle ß in the xz plane

Rotate around Y



$R_y(\beta)$

Rotation around arbitrary axis, summary:

The axis to rotate around is given as two points, $\mathbf{p}_1$ and $\mathbf{p}_2$.

$\mathbf{v} = \mathbf{p}_2 - \mathbf{p}_1$

$\mathbf{u} = \mathbf{v} / |\mathbf{v}| = (u_x, u_y, u_z)$ Normalized!

$d = \sqrt{u_y^2 + u_z^2})$

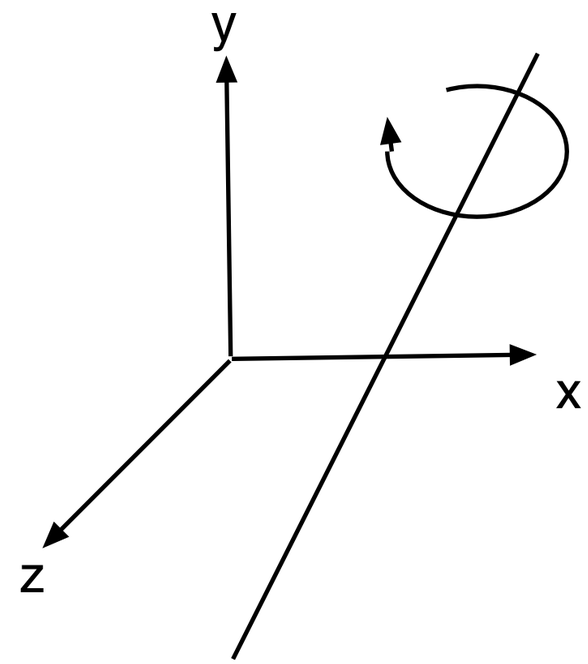
$$\mathbf{R}_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & u_z/d & -u_y/d & 0 \\ 0 & u_y/d & u_z/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_y = \begin{bmatrix} d & 0 & -u_x & 0 \\ 0 & 1 & 0 & 0 \\ u_x & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Total transformation:

$$\mathbf{R}(\theta) = \mathbf{T}(\mathbf{p}_1) * \mathbf{R}_x^T * \mathbf{R}_y^T * \mathbf{R}_z(\theta) * \mathbf{R}_y * \mathbf{R}_x * \mathbf{T}(-\mathbf{p}_1)$$

Rotation around arbitrary axis in OpenGL



Create matrices, multiply on CPU, upload to uniform matrices.

Rotation around arbitrary axis, using change of basis:

$\mathbf{v} = \mathbf{p}_2 - \mathbf{p}_1$

$\mathbf{u_z} = \mathbf{u} = \mathbf{v} / |\mathbf{v}| = (u_x, u_y, u_z)$ Normalized!

$\mathbf{u_y} = \mathbf{u} \times (u_x,0,0) / |\, \mathbf{u} \times (u_x,0,0) \,|$

$\mathbf{u_x} = \mathbf{u_y} \times \mathbf{u_z}$

$$\mathbf{R} = \begin{bmatrix} u_{x1} & u_{x2} & u_{x3} & 0 \\ u_{y1} & u_{y2} & u_{y3} & 0 \\ u_{z1} & u_{z2} & u_{z3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Total transformation:

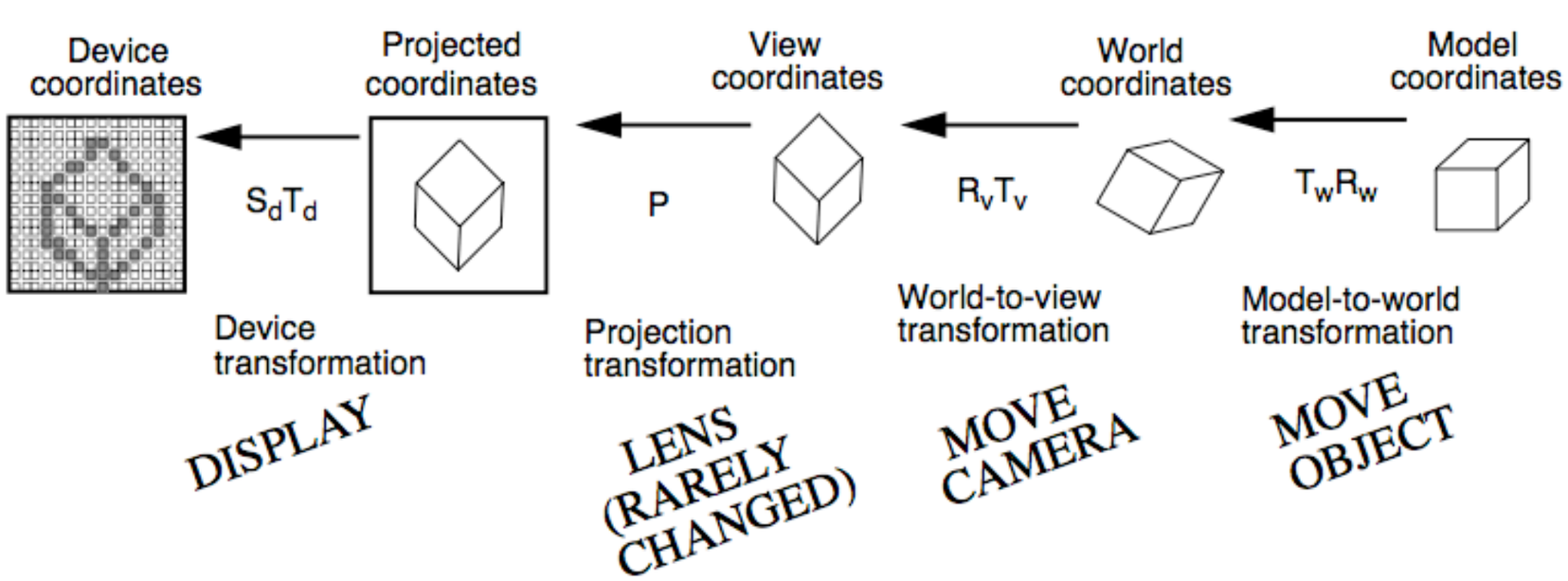
$$\mathbf{R(\theta) = T(p_1) * R^T * R_z(\theta) * R * T(-p_1)}$$

Application of rotation around arbitrary axis: Trackball control

Create rotation from user's mouse input

Picking for object selection

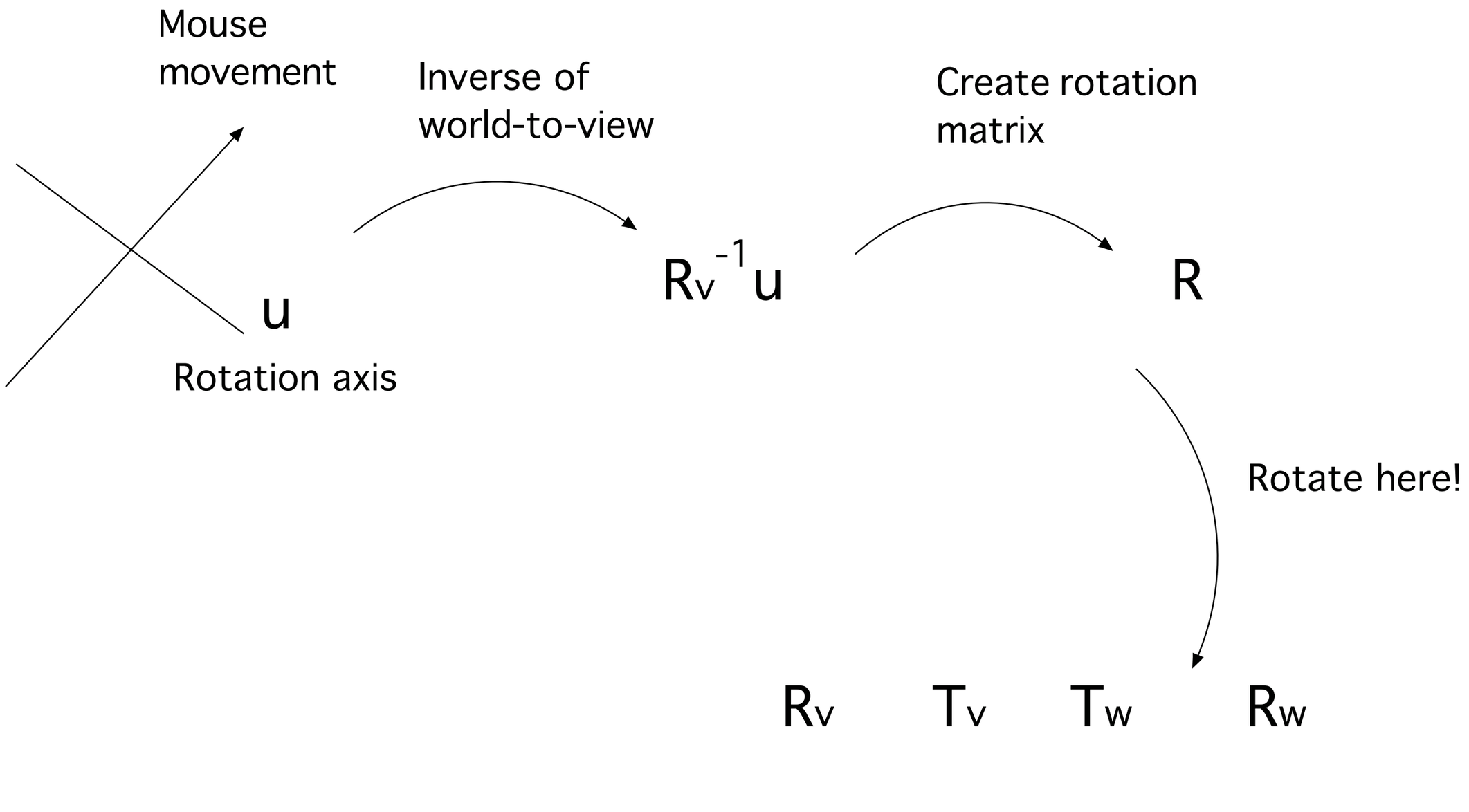
Trackball: What coordinate system?



Input on screen -> view coordinates

Rotation of model -> rotate near model

Solution: Transform to model coordinates, rotations only (avoid rotation of translation)


Mouse
movement
Inverse of
world-to-view
Create rotation
matrix
u
$R_v^{-1}u$
R
Rotation axis
Rotate here!
R_v
T_v
T_w
R_w

Pretty easy, just rotation around arbitrary axis and knowing the transformation sequence!