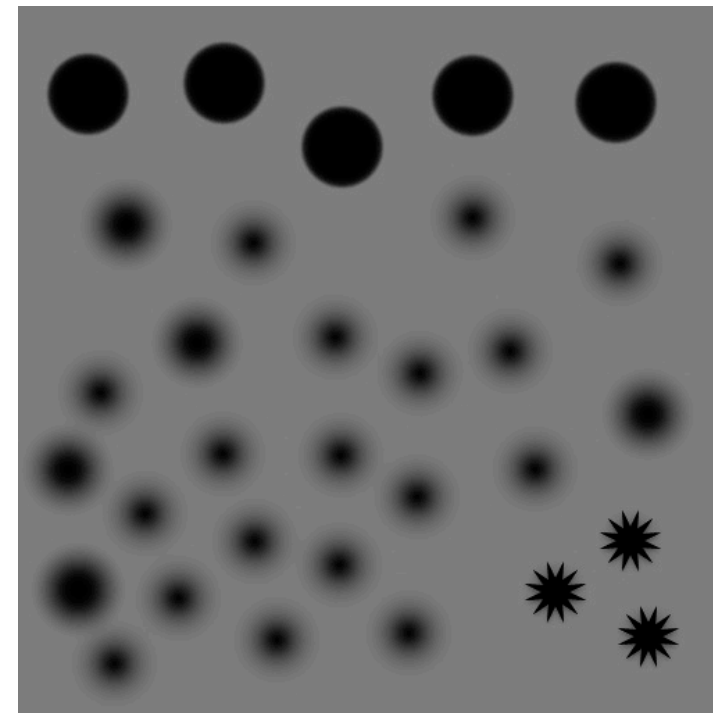
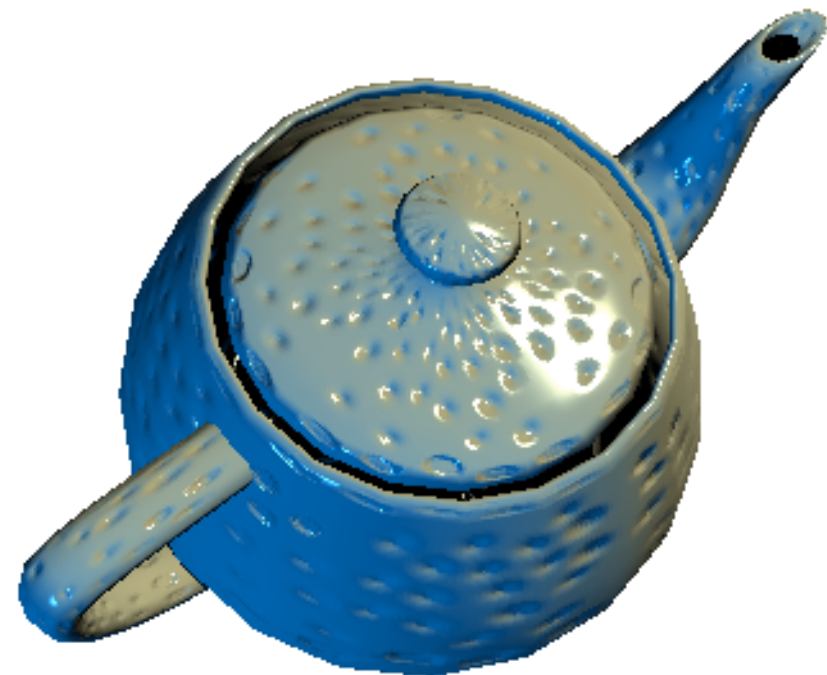




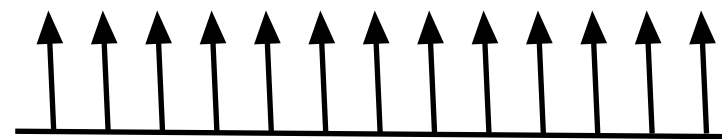
Bump mapping

Simulates surface structure by manipulating the normal vector





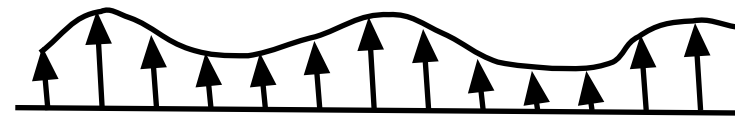
Bump mapping - model



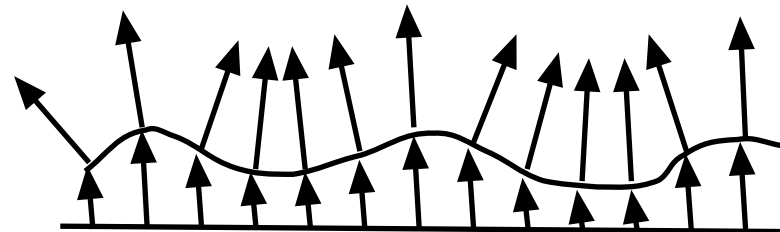
Surface with normal vectors



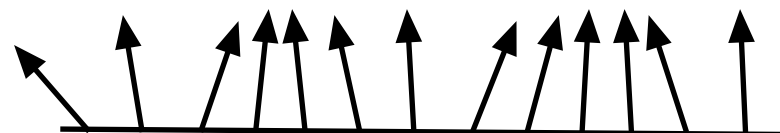
Bump map: scalar function of the texture coordinates



Modulate the surface by the bump function, along normal



Calculate new normals



Resulting normal vectors



Bump mapping - the coordinate systems

Input:

A point \mathbf{p} , normal vector \mathbf{n}

Texture coordinates $s(\mathbf{p})$, $t(\mathbf{p})$

Directions of texture coordinates \mathbf{s} , \mathbf{t}

The bump function $b(s,t)$

Calculate the partial derivative of the bump function, b_s and b_t

$$\mathbf{n}' = \mathbf{n} + b_t * (\mathbf{s} \times \mathbf{n}) + b_s * (\mathbf{t} \times \mathbf{n})$$

or, if \mathbf{s} , \mathbf{t} , \mathbf{n} are orthogonal

$$\mathbf{n}' = \mathbf{n} + b_s * \mathbf{s} + b_t * \mathbf{t}$$



Texture coordinate system

How do we find the s and t vectors? We have the texture coordinates but no coordinate system!

Cross product with normal vector? With what?



Faking it

Cross product with absolutely anything!

$$\mathbf{s} = \mathbf{x} \times \mathbf{n} / |\mathbf{x} \times \mathbf{n}|$$
$$\mathbf{t} = \mathbf{n} \times \mathbf{s}$$

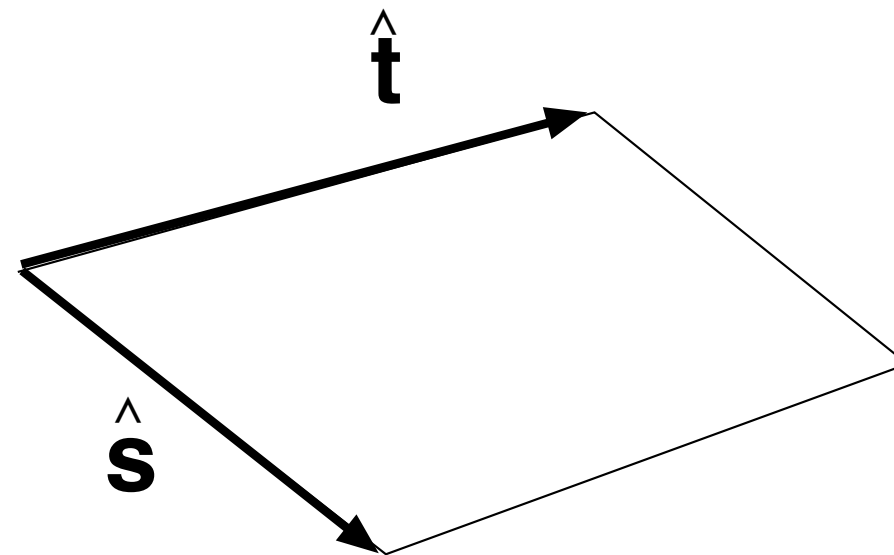
Works for some cases. (Noise bump maps in particular.)

But we can do better!



Trivial geometry

Very easy for a cube. Comfortable test case.





Lengyel's method

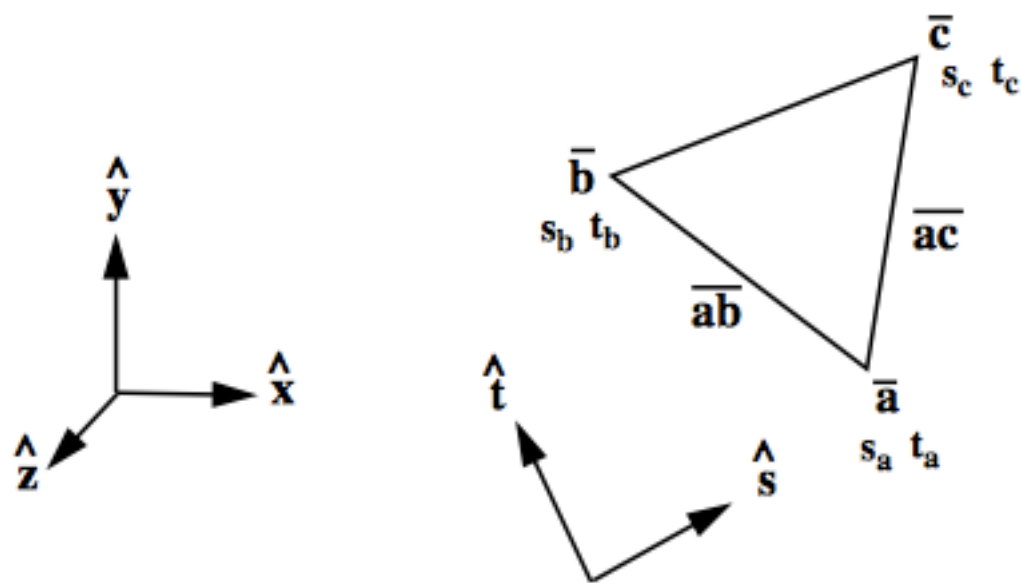
Derive through steps by s and t in xyz space

Straight and clean method using matrix algebra

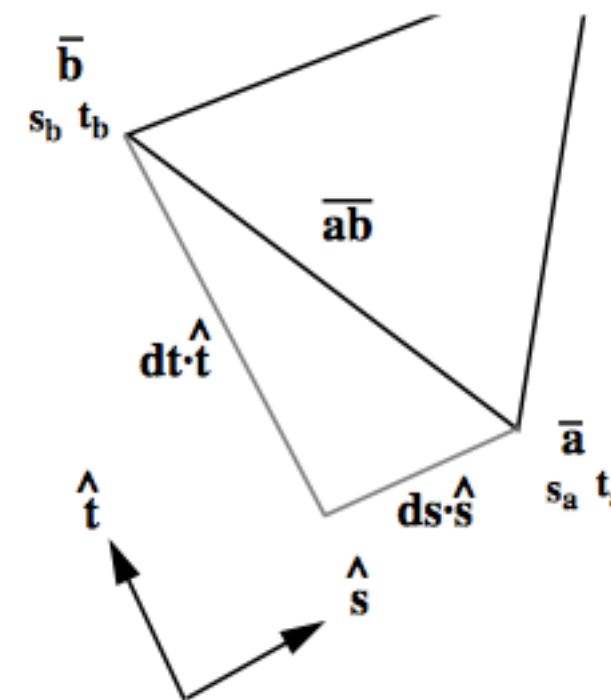
**Express two line segments as function of \hat{s} and \hat{t} ,
find the inverse!**



Lengyel's method



Given a triangle with texture coordinates, find basis vectors for texture coordinates!



Take edge ab , split to components along s and t . Express as matrix. Find s and t by matrix inverse!



Lengyel's method

in program code - fairly simple!

```
float ds1 = sb - sa; float ds2 = sc - sa;  
float dt1 = tb - ta; float dt2 = tc - ta;  
vec3 s, t;  
float r = 1/(ds1 * dt2 - dt1 * ds2);  
s = (ab * dt2 - ac * dt1) * r;  
t = (ac * ds1 - ab * ds2) * r;
```

Note! Vector operations!



Approximative method

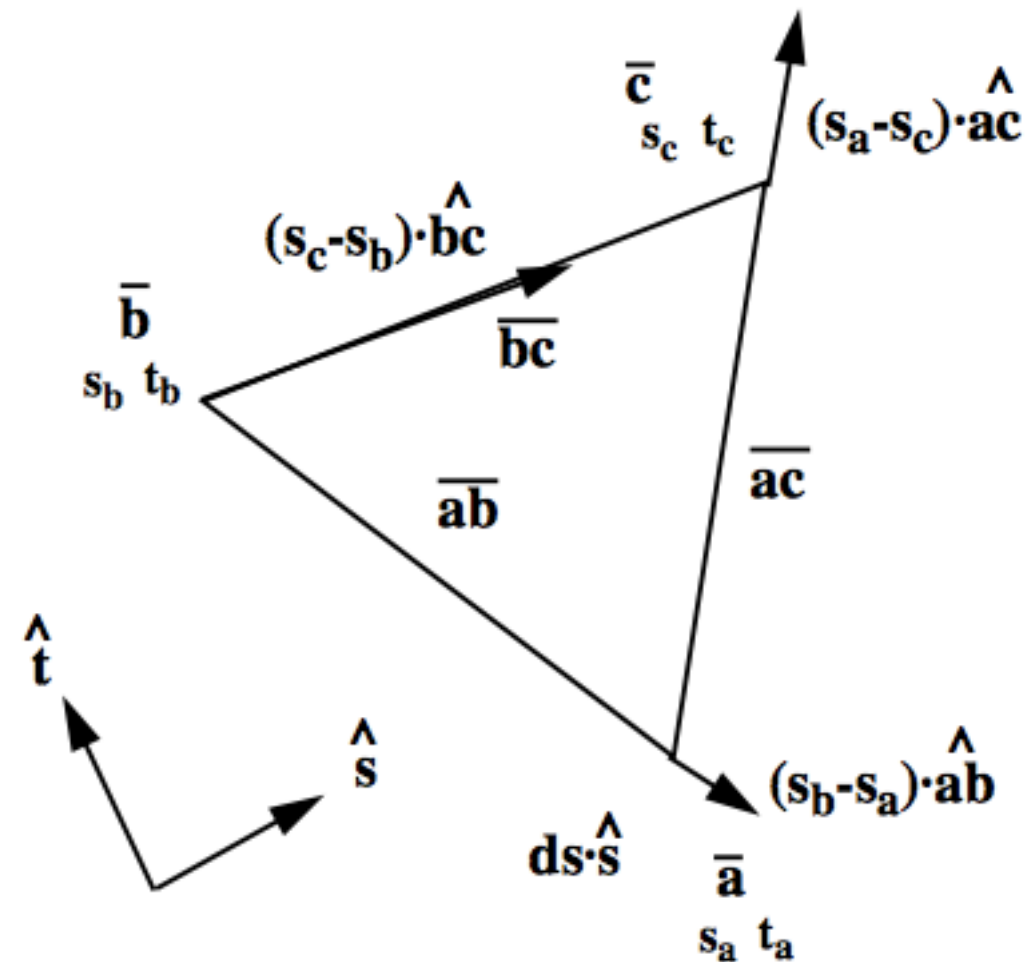
Let each edge of a polygon contribute to s and t depending on their variation in s and t !

Contribution to s from each edge = the edge direction normalized times the variation in s .



Approximative method

$$s_{ab} = \frac{ab}{|ab|} (s_b - s_a)$$





Both methods give good results for complicated models!

