TSBK 07

Computer Graphics

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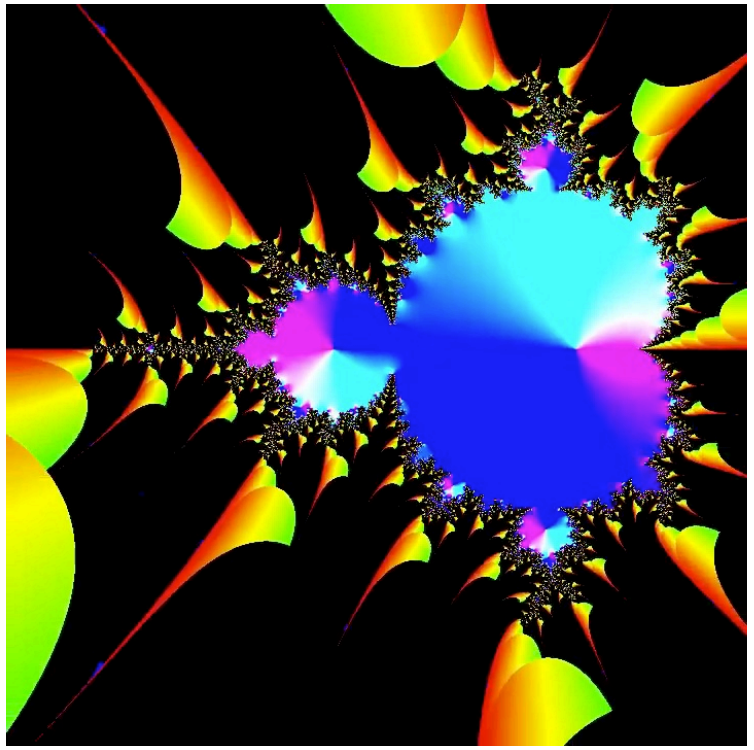

Lecture 2

2D transformations

Introduction to OpenGL

Fundamental vector operations

A vector is *positional* or *directional.*

Vector addition: $\mathbf{a} + \mathbf{b} = (a_x+b_x, a_y+b_y, a_z+b_z)$

Multiplication with scalar: $s * \mathbf{a} = (s*a_x, s*a_y, s*a_z)$

Magnitude: $|\mathbf{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Normalize: $\hat{\mathbf{a}} = |\mathbf{a}|^{-1} * \mathbf{a}$

Dot product

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}| * |\mathbf{b}| * \cos\theta = a_x b_x + a_y b_y + a_z b_z$

Properties of dot products:

Scalar value!

$\mathbf{a}\cdot\mathbf{b} = 0$ if a and b are orthogonal

$\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$ Commutative

$\mathbf{a}\cdot(\mathbf{b} + \mathbf{c}) = \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\cdot\mathbf{c}$ Distributive

Cross product

$$\mathbf{a} \times \mathbf{b} = \hat{\mathbf{n}} * |\mathbf{a}| * |\mathbf{b}| * \sin\theta =$$

$$(a_y b_z - a_z b_y,\ a_z b_x - a_x b_z,\ a_x b_y - a_y b_x)$$

Properties of cross products:

Vector!

Orthogonal to both **a** and **b**

$\mathbf{a} \times \mathbf{b} = (0,0,0)$ if **a** and **b** are parallel

$\mathbf{a} \times \mathbf{b} = - \mathbf{b} \times \mathbf{a}$ Non-Commutative

$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ Distributive

Matrix-vector multiplication

$$\mathbf{M} * \mathbf{a} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} * \begin{bmatrix} a_x \\ a_y \end{bmatrix} =$$

$$= \begin{bmatrix} M_{11}a_x + M_{12}a_y \\ M_{21}a_x + M_{22}a_y \end{bmatrix}$$

Matrix-matrix multiplication

$$\mathbf{M} * \mathbf{N} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} * \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$$

Properties of matrix multiplications:

Associative:

A*B*C = (A*B)*C = A*(B*C)

Non-commutative:

A*B and B*A not guaranteed to be equal!

Identity matrix:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Zeroes everywhere except on diagonal, which is 1

IA = A

The ”one” of matrices

Matrices may have any size, 2x2, 3x3, 4x1, 100x2...

A vector is also a single-column matrix!

We mainly care about 3x3 and 4x4 matrices in this course!

Some operations on matrices:

Inverse $AA^{-1} = I$

Transpose A^T

Dot product = matrix multiplication of a row and a column matrix!