

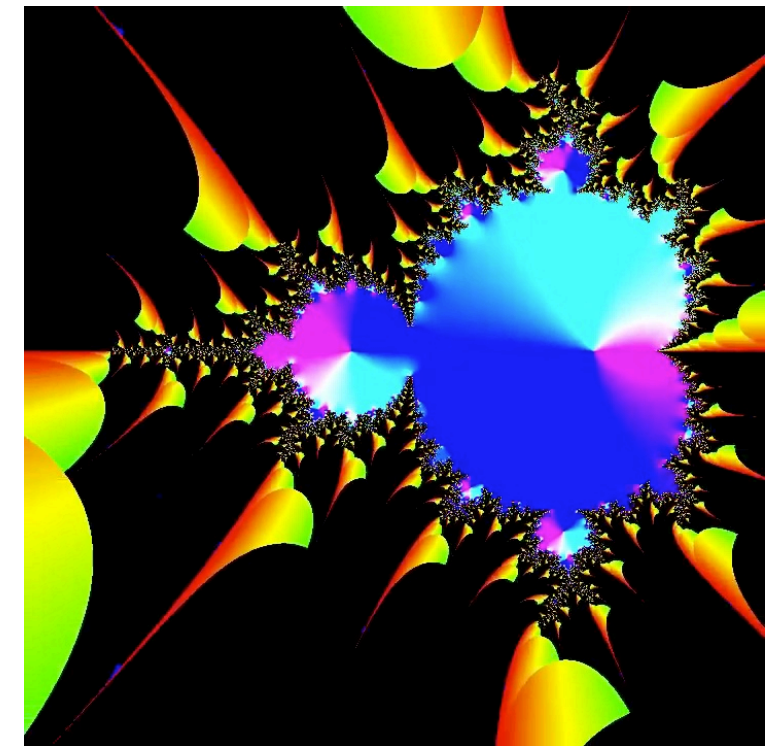


Information Coding / Computer Graphics, ISY, LiTH

TSBK 07

Computer Graphics

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Lecture 2

2D transformations

Introduction to OpenGL



Fundamental vector operations

A vector is *positional* or *directional*.

Vector addition: $\mathbf{a} + \mathbf{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$

Multiplication with scalar: $s * \mathbf{a} = (s * a_x, s * a_y, s * a_z)$

Magnitude: $|\mathbf{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Normalize: $\hat{\mathbf{a}} = |\mathbf{a}|^{-1} * \mathbf{a}$



Dot product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| * |\mathbf{b}| * \cos \theta = a_x b_x + a_y b_y + a_z b_z$$

Properties of dot products:

Scalar value!

$\mathbf{a} \cdot \mathbf{b} = 0$ if \mathbf{a} and \mathbf{b} are orthogonal

$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$ Commutative

$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$ Distributive



Cross product

$$\mathbf{a} \times \mathbf{b} = \hat{\mathbf{n}} * |\mathbf{a}| * |\mathbf{b}| * \sin \theta =$$

$$(a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)$$

Properties of cross products:

Vector!

Orthogonal to both **a** and **b**

a × **b** = (0,0,0) if **a** and **b** are parallel

a × **b** = - **b** × **a** Non-Commutative

a × (**b** + **c**) = **a** × **b** + **a** × **c** Distributive



Matrix-vector multiplication

$$\mathbf{M} * \mathbf{a} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} * \begin{bmatrix} a_x \\ a_y \end{bmatrix} =$$
$$= \begin{bmatrix} M_{11}a_x + M_{12}a_y \\ M_{21}a_x + M_{22}a_y \end{bmatrix}$$

Matrix-matrix multiplication

$$\mathbf{M} * \mathbf{N} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} * \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$$



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Properties of matrix multiplications:

Associative:

$$A*B*C = (A*B)*C = A*(B*C)$$

Non-commutative:

$A*B$ and $B*A$ not guaranteed to be equal!



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Identity matrix:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Zeros everywhere except on diagonal, which is 1

$$IA = A$$

The "one" of matrices



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Matrices may have any size, 2×2 , 3×3 , 4×1 , 100×2 ...

A vector is also a single-column matrix!

We mainly care about 3×3 and 4×4 matrices in this course!



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Some operations on matrices:

Inverse $AA^{-1} = I$

Transpose A^T

Dot product = matrix multiplication of a row and a column matrix!