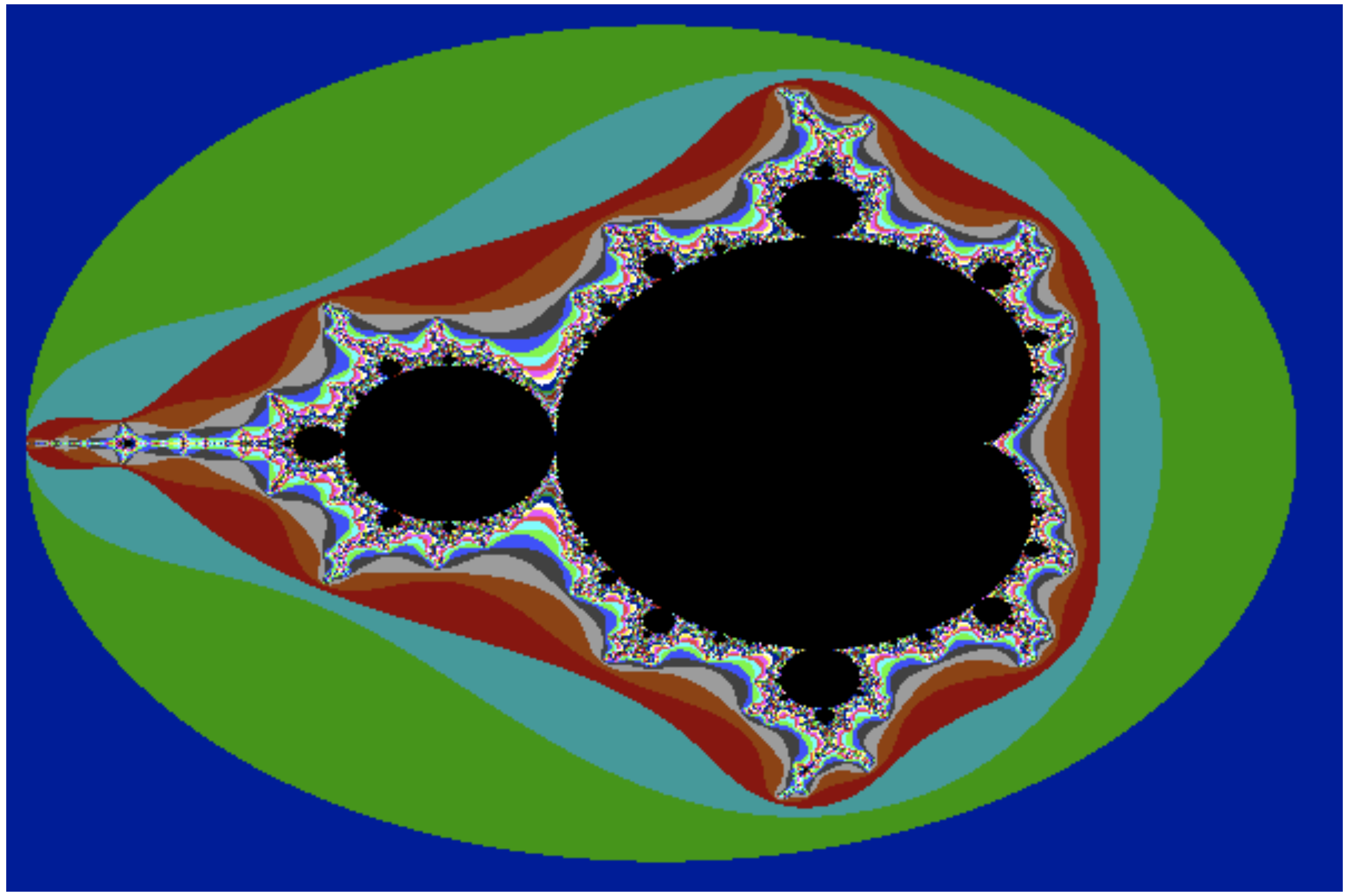
Fractals, noise and procedural modelling

Creating complex and interesting shapes from code

Most famous fractal: Mandelbrot set

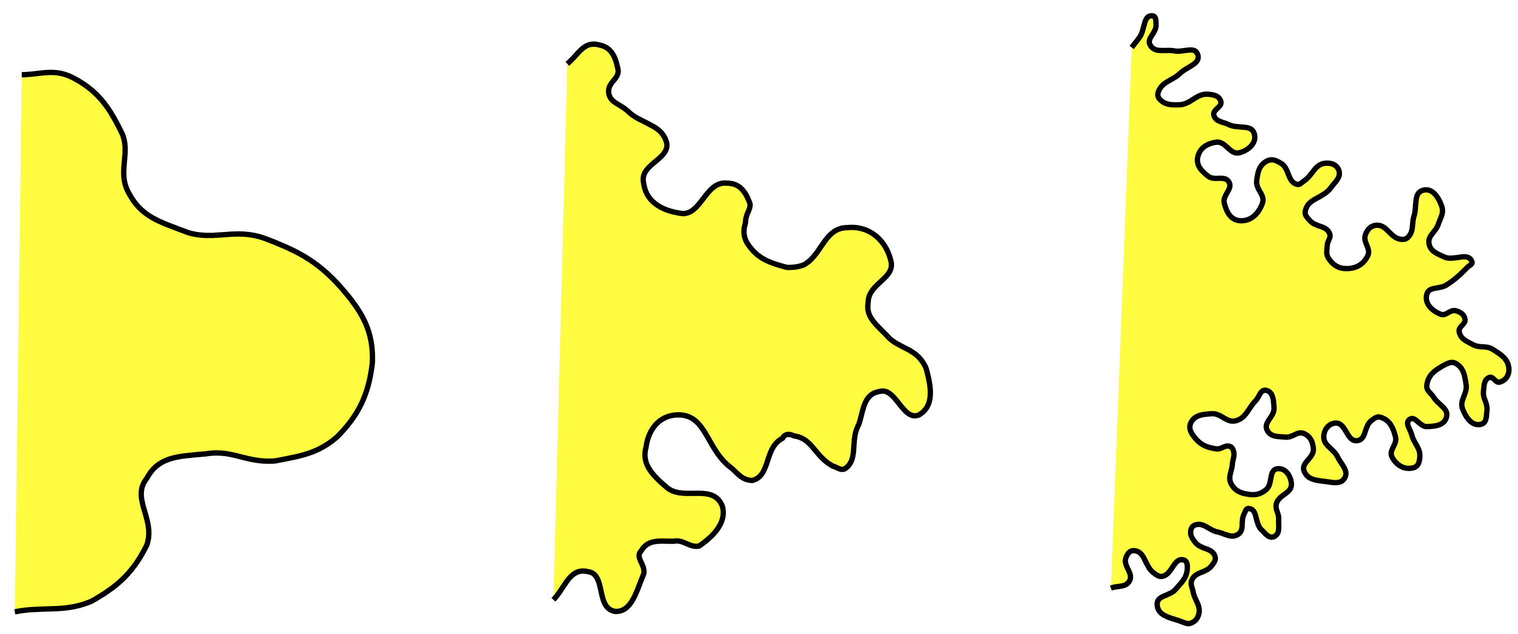


What is it, more than a pretty image?

Natural objects have fractal features

Classic example: Coastline

Shape and length varies with resolution



Fractals in computer graphics

Fractals are shapes with:

- **self-similarity**
- **infinite resolution**

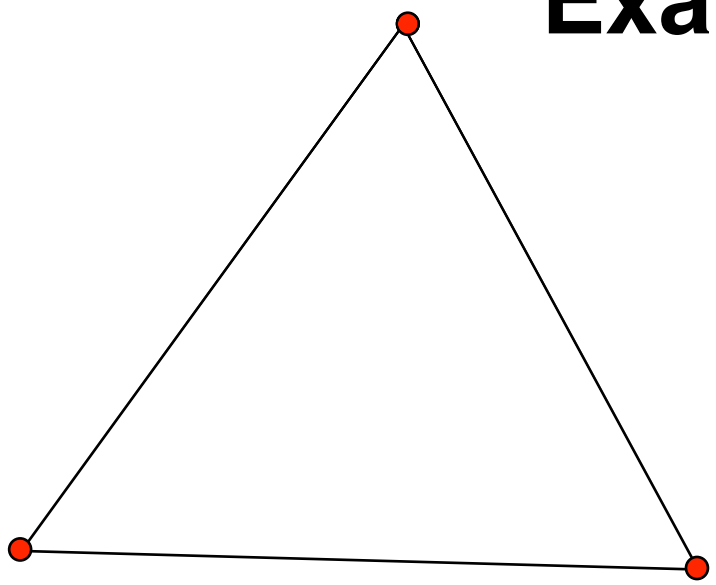
Used for modelling such shapes

Classification of fractals

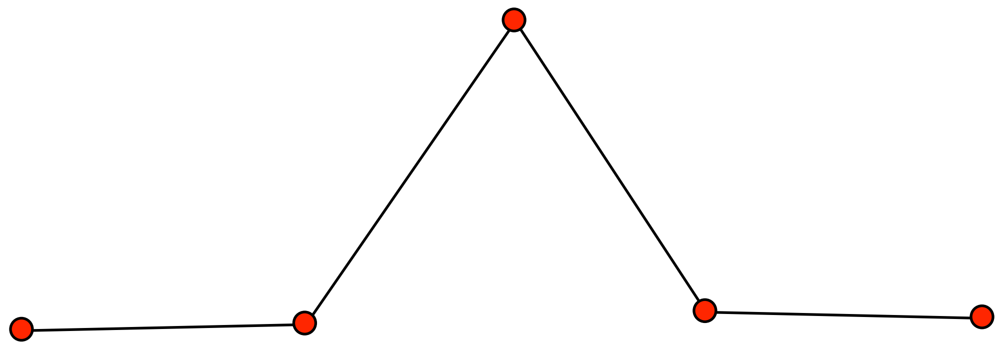
- geometrical recursive construction
- stochastic fractals
- mathematical formulas (in the complex plane)

Geometric construction of self-simlar fractals

Example: Koch curve

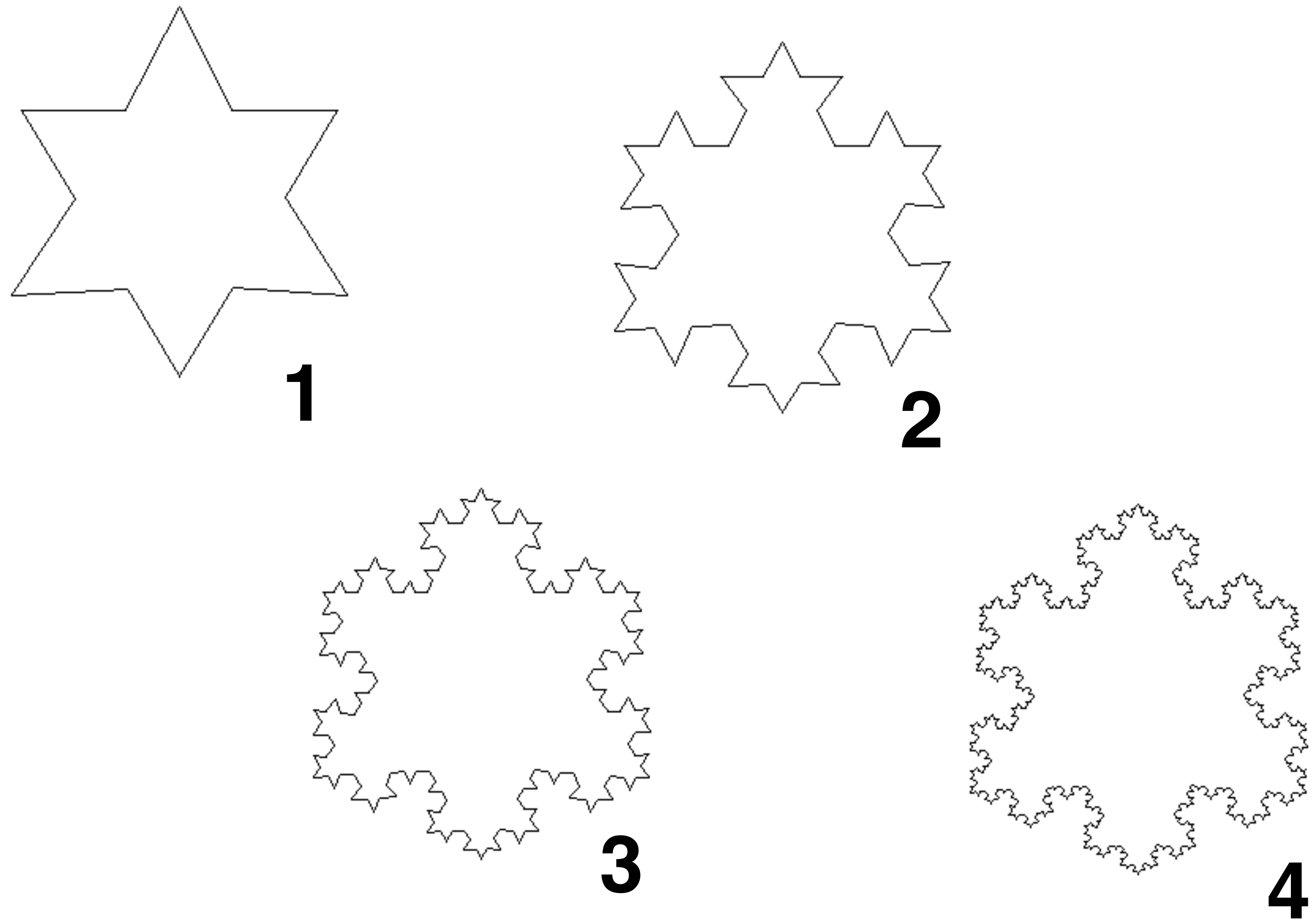


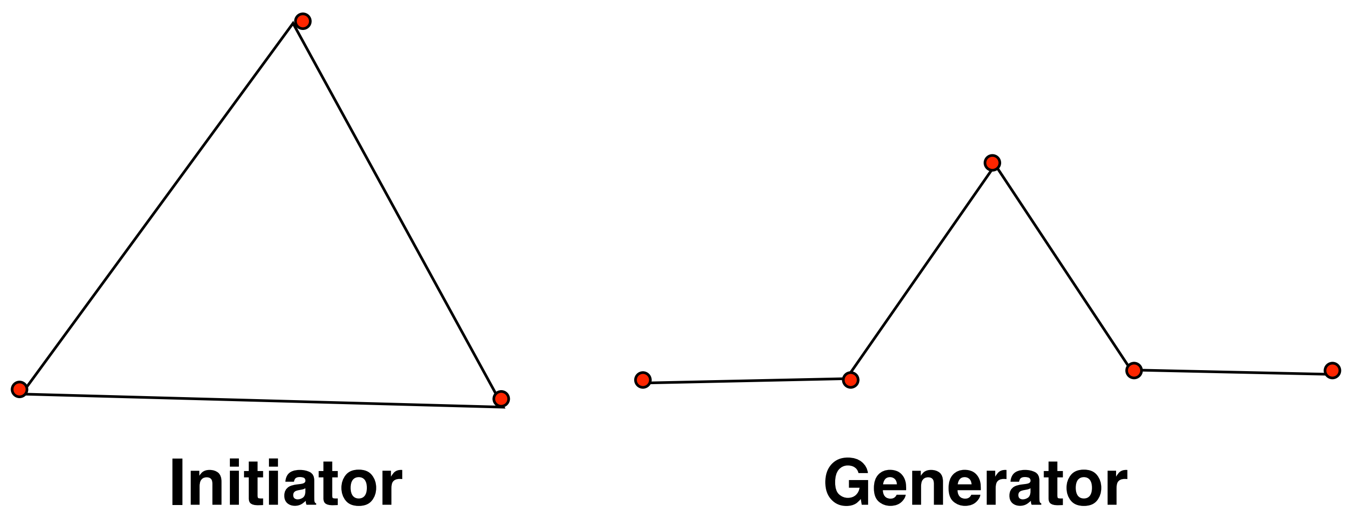
Initiator



Generator

Resulting Koch curves





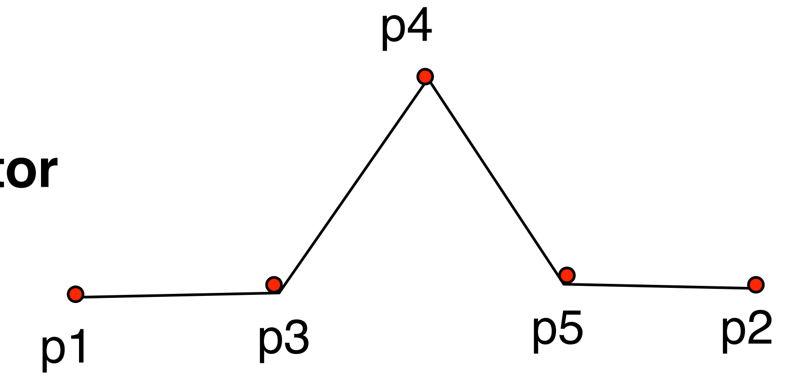
Recursive function

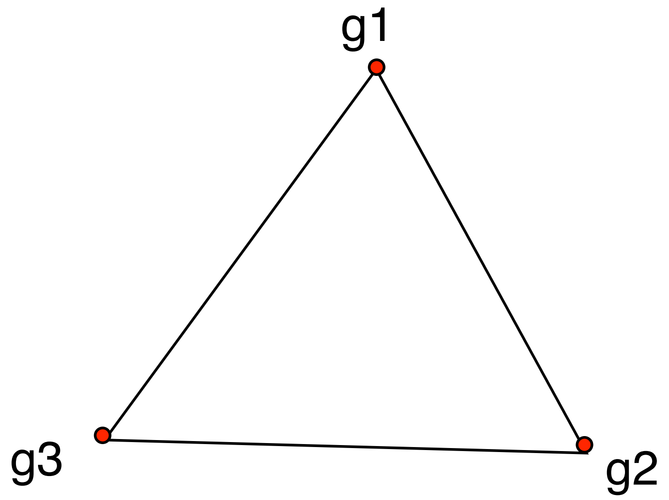
Pass all parts to next level

Replace part with the generator, scaled to same length

Stop at desired recursion depth or when sections are small enough (e.g. 1 pixel long)

```
procedure DrawKoch(p1, p2, depth)

if depth >= maxDepth then

 MoveTo(p1)
 LineTo(p2)
 return

else

 calculate p3, p4, p5 as the three points inside the generator

 DrawKoch(p1, p3, depth+1)
 DrawKoch(p3, p4, depth+1)
 DrawKoch(p4, p5, depth+1)
 DrawKoch(p5, p2, depth+1)


main procedure:

Choose three generator points, g1, g2, g3

DrawKoch(g1, g2, 0)
DrawKoch(g2, g3, 0)
DrawKoch(g3, g1, 0)
```





Fractal dimension

A measure of how rough or fragmented the shape is

Definition:

$$ns^D = 1$$

n = number of subparts
s = scaling
D = fractal dimension

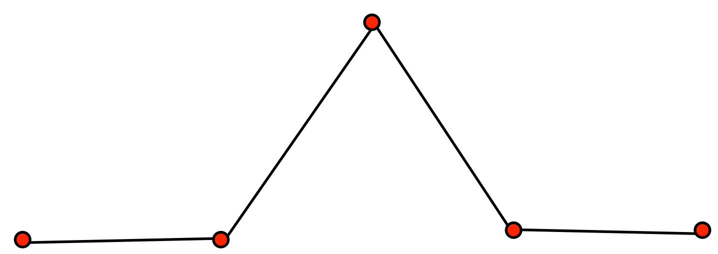
Solves to D = ln(n) / ln (1/s)

Fractal dimension example:

Koch curve

n = 4

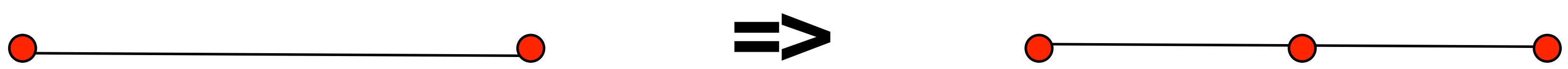
s = 1/3



D = ln 4 / ln 3 = 1.26

Fractal dimension example:

Splitting a line



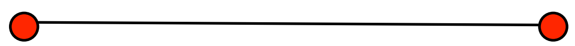
$$n = 2$$

$$s = 1/2$$

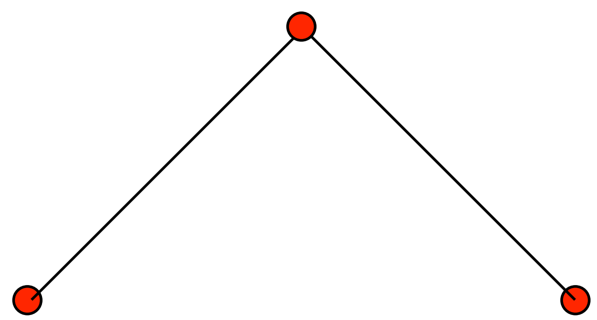
$$D = \ln 2 / \ln 2 = 1$$

Fractal dimension example:

Splitting a line and moving midpoint



=>



$$n = 2$$

$$s = 1/\sqrt{2}$$

$$D = \ln 2 / \ln \sqrt{2} = 2$$

Fractal dimension:

In 2D:

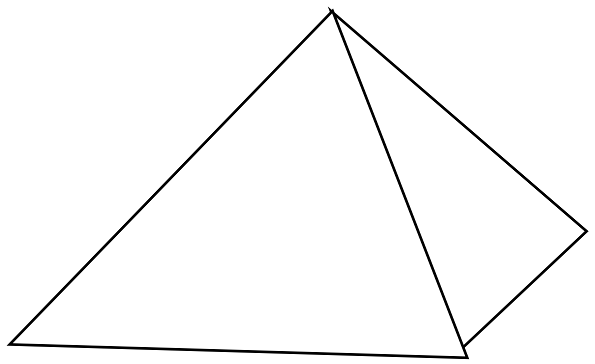
1 to 2: Well-behaved fractal curve

>2: Self-intersecting, area-covering

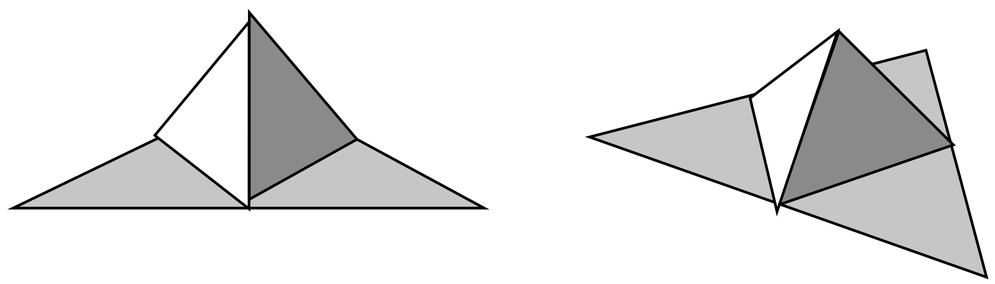
Split line: D = 1 minimum, no fractal
Koch: D = 1.26, moderate fractal
Moved midpoint: D = 2, maximum

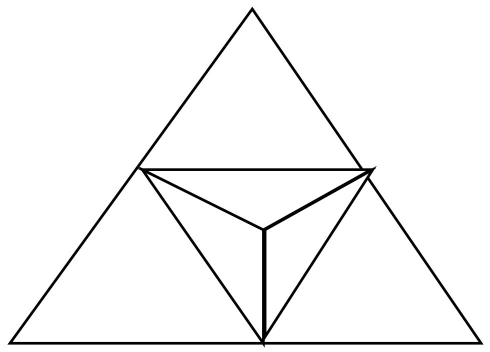
Geometric construction of self-similar fractals in 3D

Example



Initiator





Generator

n = 6

s = 1/2

D = ln 6 / ln 2 = 2.58

Interpretation of fractal dimension:

In 3D:

2 to 3: Well-behaved fractal surface

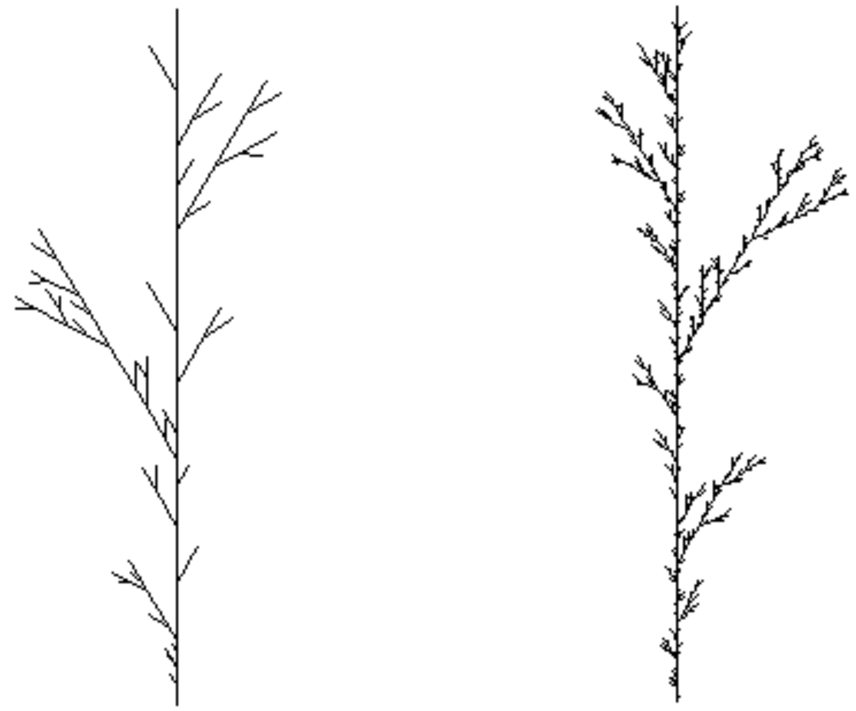
>3: Self-intersecting, volume-covering

Example: Generation of plants

Promising, but too self-similar!

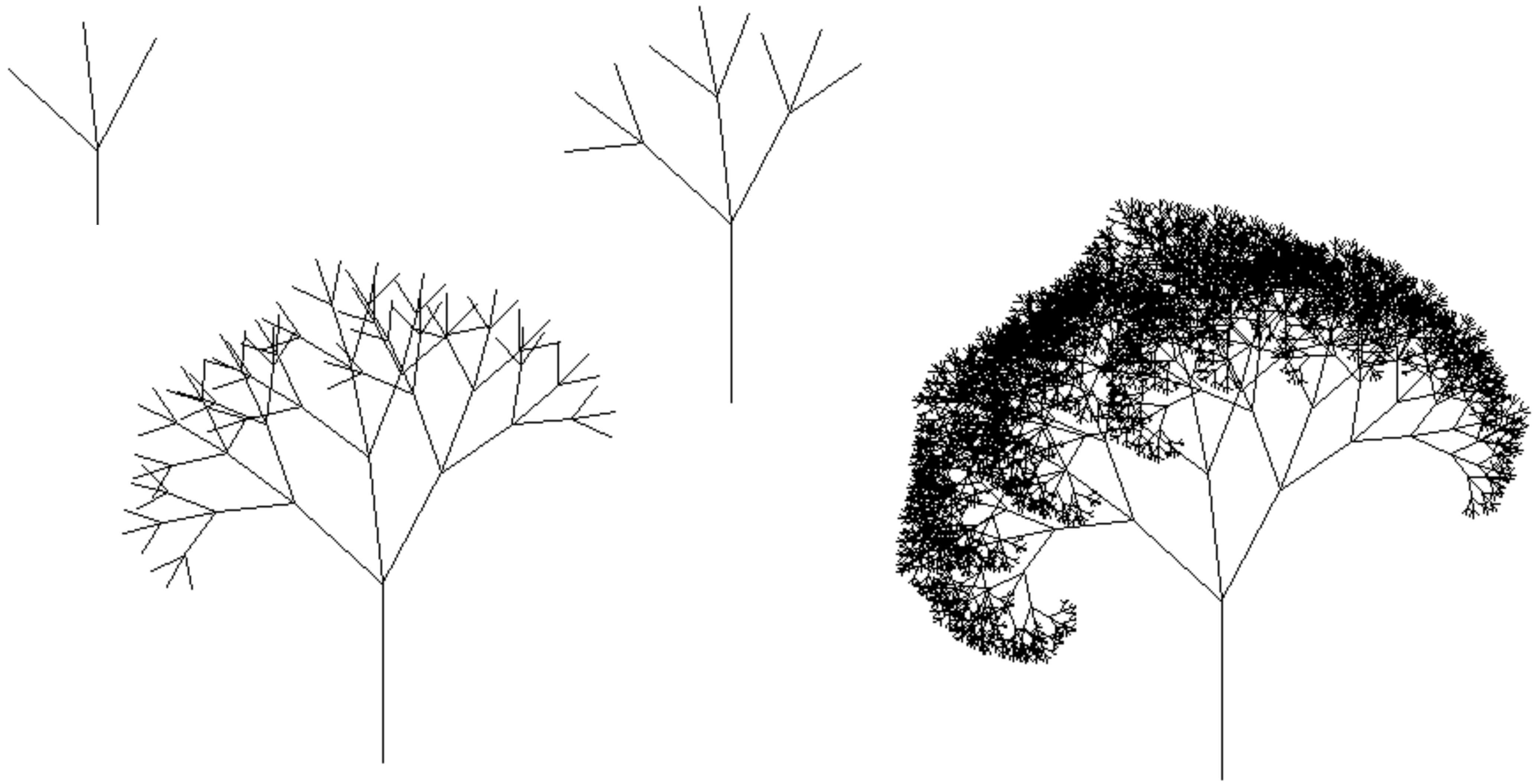
Statistically self-similar fractals

Random variation of generator



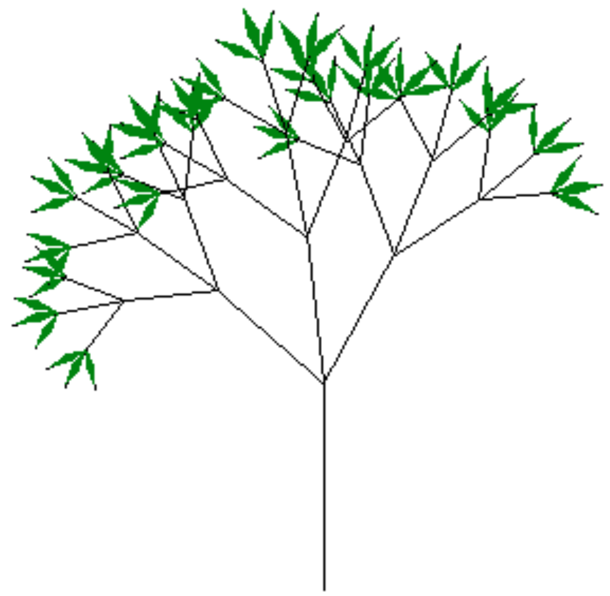
Same branch generator as before, with some randomness!

Example: Generation of plants #2



Related methods:

Shape grammars and procedural methods



No unlimited resolution

Different rules at different levels

Example: Tree with leaves: replace last iteration with leaf generator

“graftals”