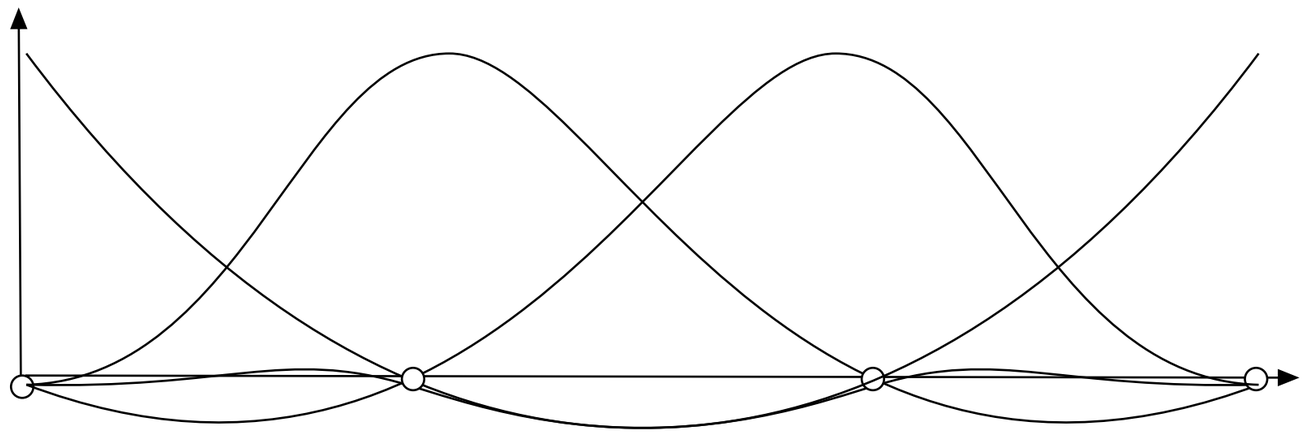
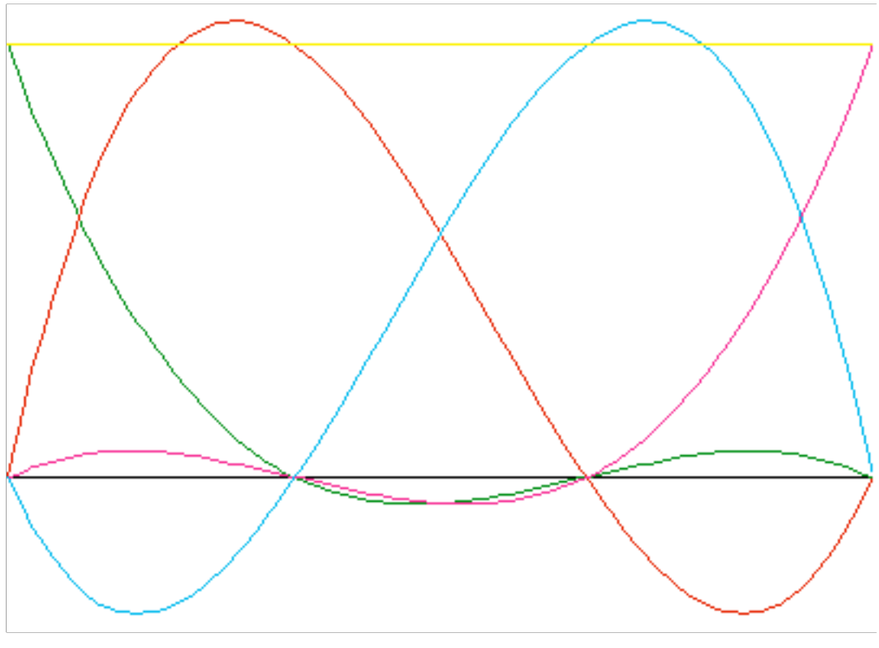
Blending functions for interpolation spline

The points are *blended* together using blending functions



Blending functions for interpolation spline

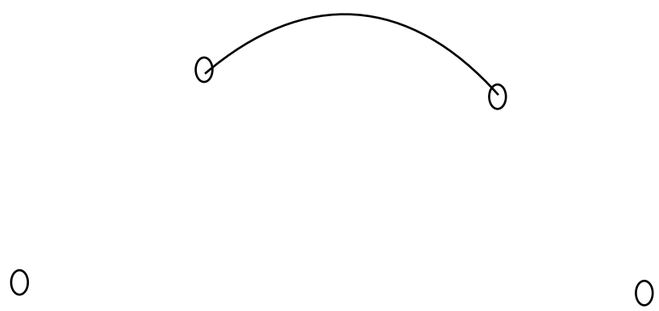
All blending functions are zero or 1 at the control points!



Actual blending functions for interpolated spline of 4 control points (similar to Bézier)

Cardinal splines
Catmull-Rom splines

Interpolation spline



Specified *only* by control points

Calculated from 4 control points, define between the middle two!

A tension parameter t can adjust the shape

t = 0 => Catmull-Rom

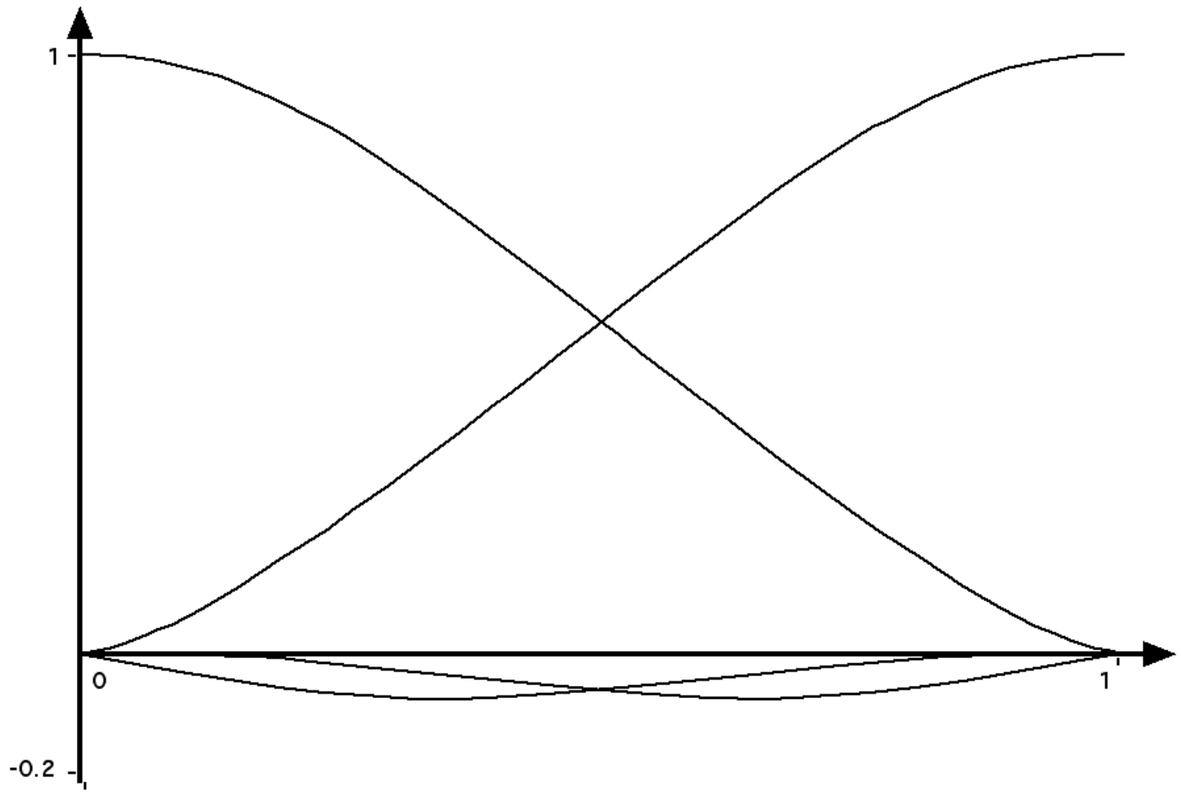
Catmull-Rom splines,
Matrix form

$$P(u) = [\, u^3 \; u^2 \; u \; 1 \,] \begin{bmatrix} -1/2 & 3/2 & -3/2 & 1/2 \\ 1 & -5/2 & 2 & -1/2 \\ -1/2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{k-1} \\ p_k \\ p_{k+1} \\ p_{k+2} \end{bmatrix}$$

$$P(u) = p_{k-1}(-u^3/2 + u^2 - u/2) + p_k(3u^3/2 - 5u^2/2 + 1) + p_{k+1}(-3u^3/2 + 2u^2 + u/2) + p_{k+2}(u^3/2 - u^2/2)$$

$$= p_{k-1}*CAR_0(u) + p_k*CAR_1(u) + p_{k+1}*CAR_2(u) + p_{k+2}*CAR_3(u)$$

Catmull-Rom splines,
Blending functions



NURBs/NURBS

Non-Uniform Rational B-spline.

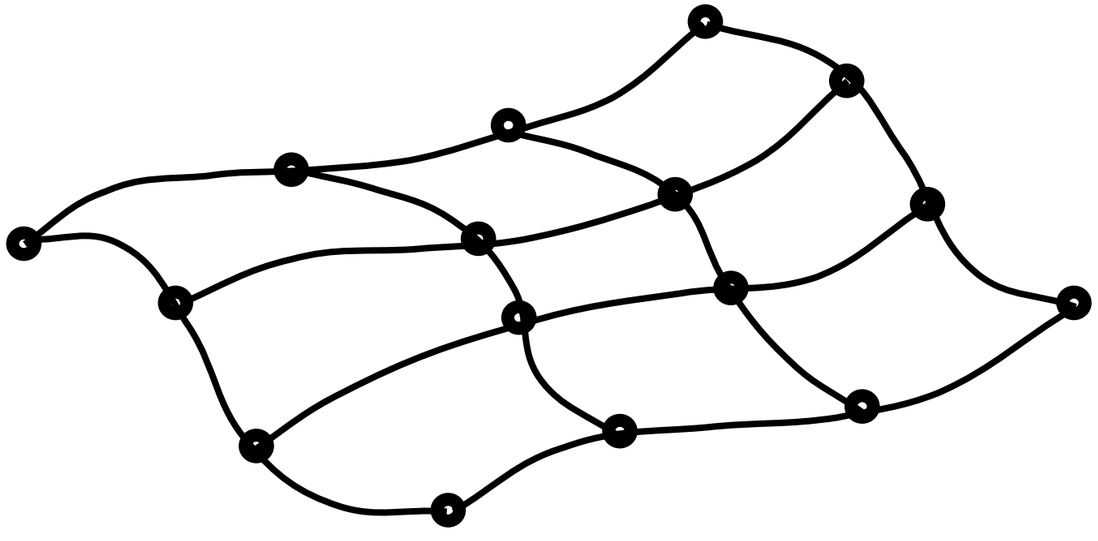
Popular in CAD programs.

Can exactly represent all quadric curves.

Bézier surfaces

A surface is built from a set of Bézier patches

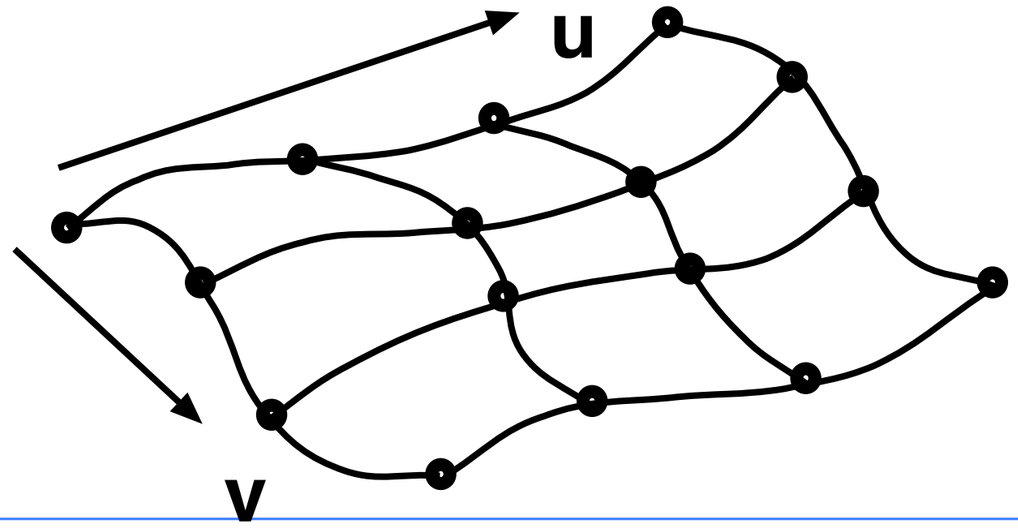
A Bézier patch consists of 16 control points in a 4x4 grid



Bézier surfaces

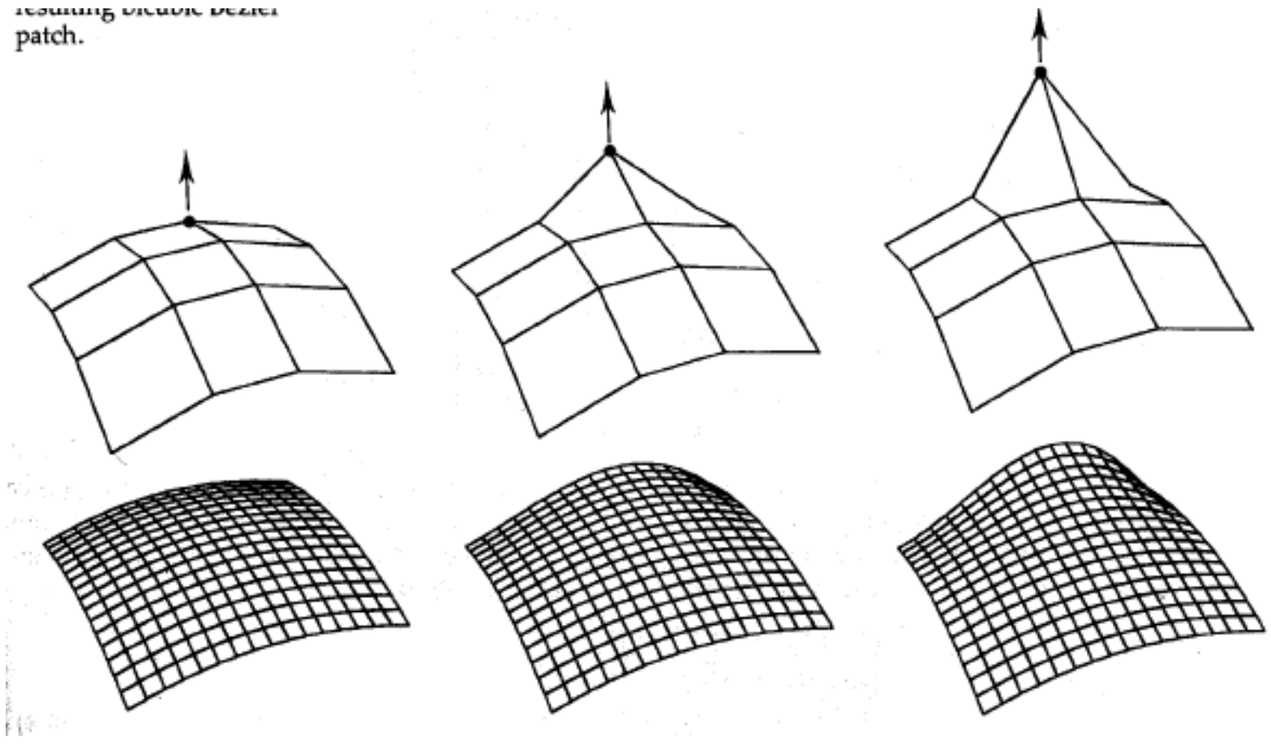
Blending of the 16 control points as a 2-dimensional sum

$$P(u,v) = \sum_{j=0}^{3} \sum_{k=0}^{3} p_{j,k}\, BEZ_{j,3}(v)\, BEZ_{k,3}(u)$$



Bézier surface example

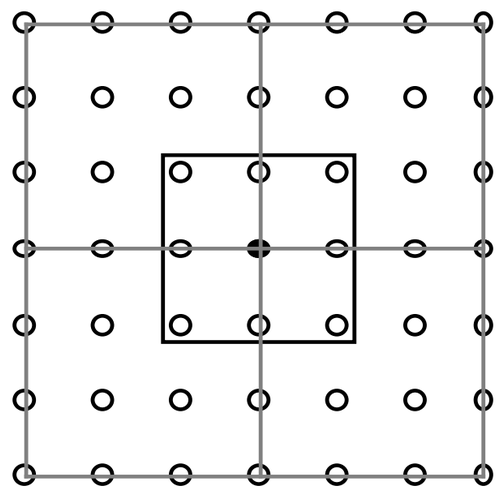
patch.



Fitting together patches

Fit in both u and v direction

Make a 3x3 “joystick” at each corner



Splines and surfaces in OpenGL

Pre-generated shapes on CPU

Generate by multi-pass GPU processing

Old OpenGL: Evauators (glMap)

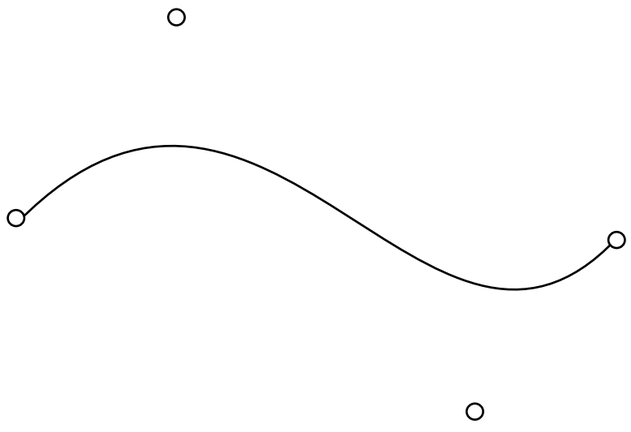
3.2: Geometry shaders

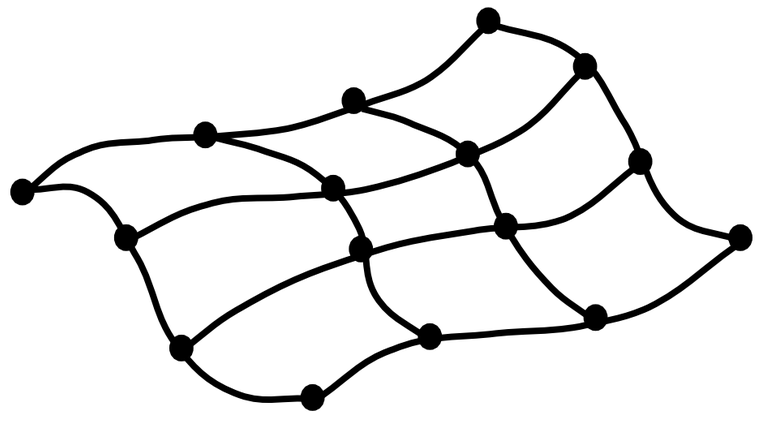
4: Tesselation shaders

Evaluators

Old built-in Bezier evaluator. Easy to use but no longer recommended.

Supported both curves and surfaces





Geometry shaders

OpenGL 3 (extension in GL 2)

Shader between vertex and fragment, converts geometry, can add new vertices

Modest hardware demand: G80 or better (2007+)

Tesselation shaders

OpenGL 4

Shader between vertex and fragment (before geometry shader), focused on subdivision

Higher hardware demand! (Southfork is OK)

Applications:

- Splines/surfaces
- Edge extraction, silhouettes
- Polygon-level effects (shrinking triangles)
- Adaptive subdivision
- Visualizing normal vectors etc

More object representation soon...

Fractals and procedural generation

but first

let us continue on with the animation subject!