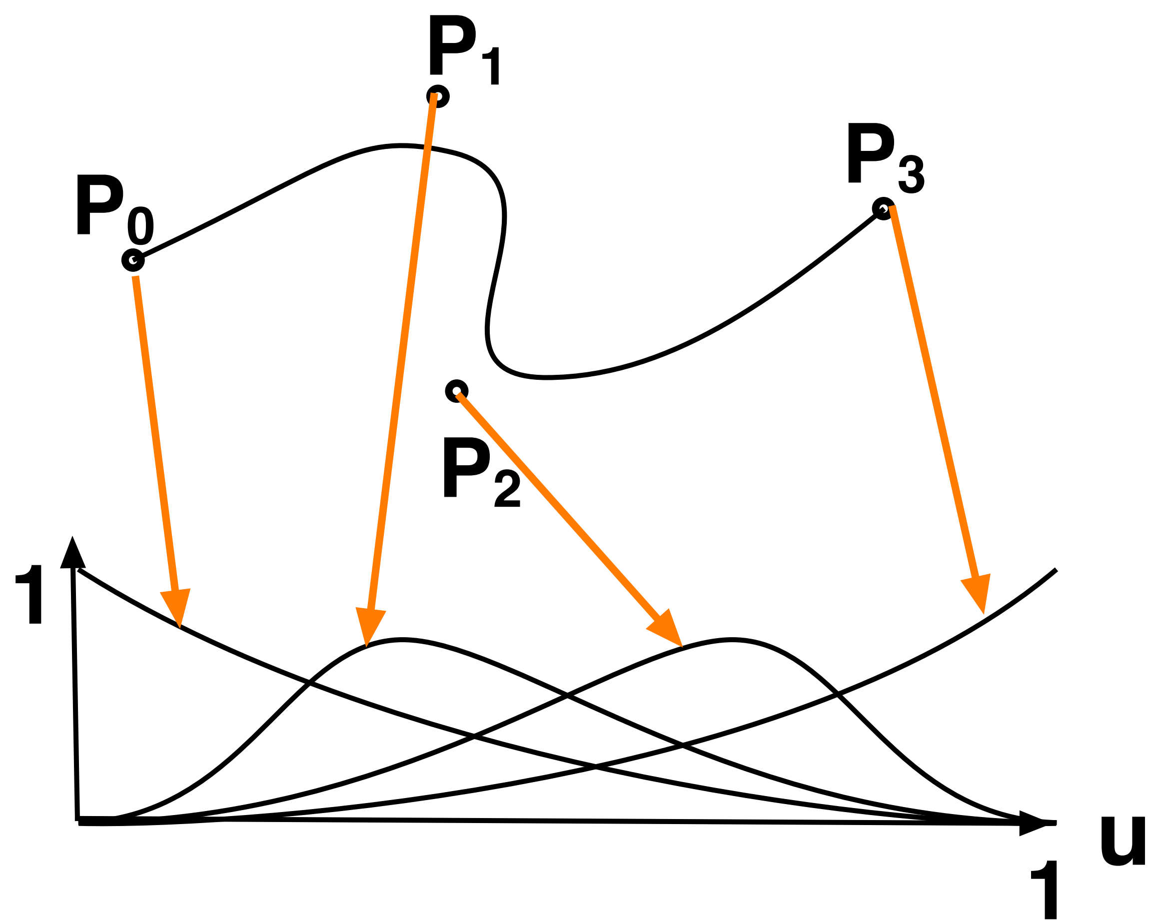
# Lecture 11

## More splines: Bézier, Catmull-Rom, Bézier surfaces

## Animation, Collision detecton

# Bézier curves




$BEZ_{0,3} = (1-u)^3$

$BEZ_{1,3} = 3u(1-u)^2u$

$BEZ_{2,3} = 3(1-u)u^2$

$BEZ_{3,3} = u^3$

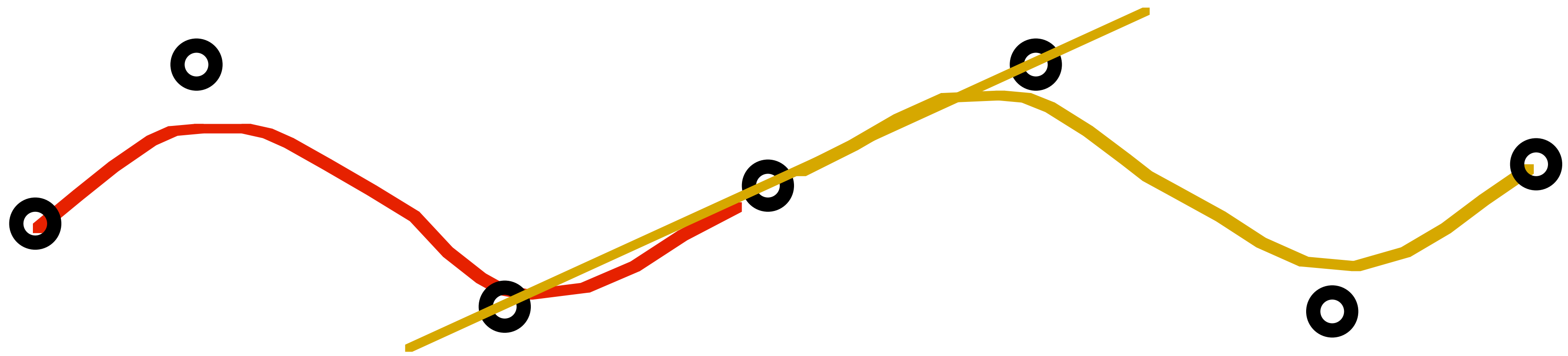
$$P(u) = P_0*(1-u)^3 + P_1*3u(1-u)^2 + P_2*3(1-u)u^2 + P_3*u^3$$

$$= \sum_{i=0}^{3} P_i * BEZ_{i,3}(u)$$

# Fitting together sections

**$C_0$/$G_0$ continuity: just fit the points**

**$C_1$ continuity: Tangents are equal along the edge.**
**$G_1$ continuity: Tangents have same direction along the edge.**
**Simple method: Put 3 points in a line**



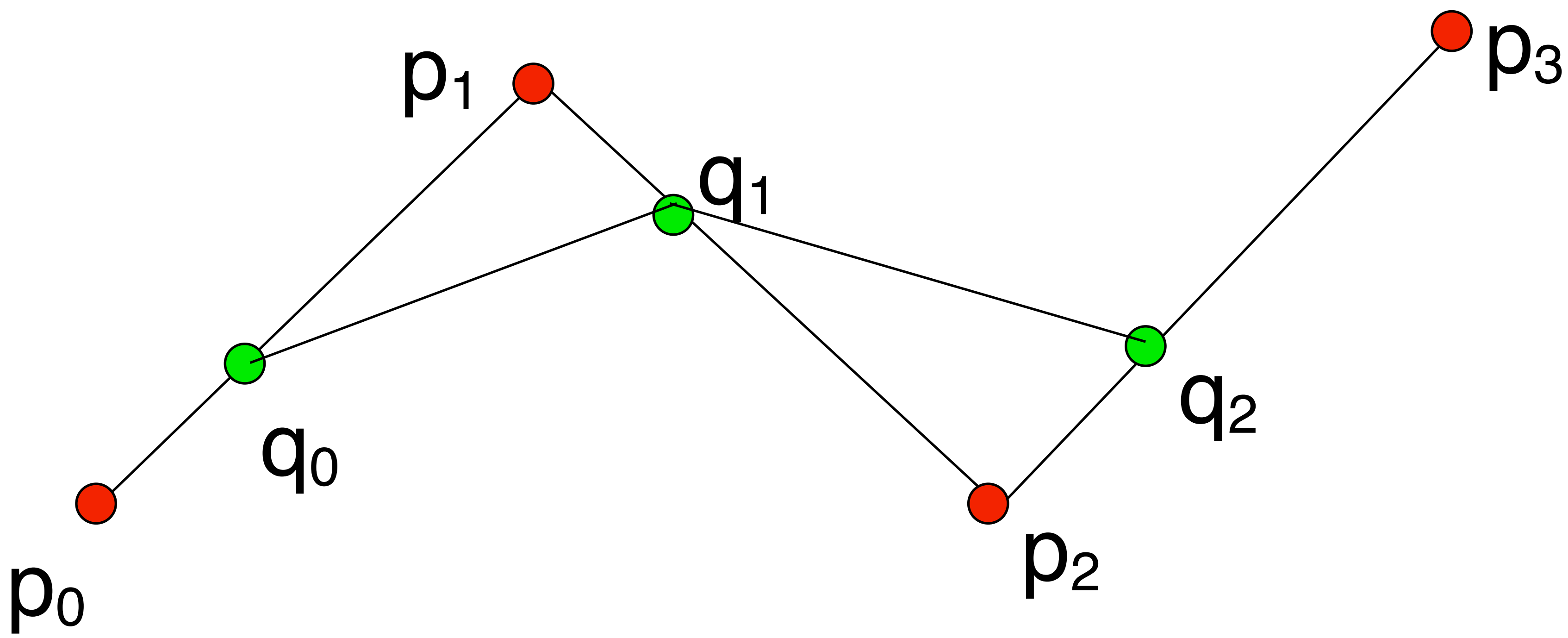
# Quadratic Bezier curves

## Three control points
## 2nd order polynomials

$$\mathbf{p}(u) = (1-u)2\mathbf{p_0} + 2u(1-u)\mathbf{p_1} + u^2\mathbf{p_2}$$

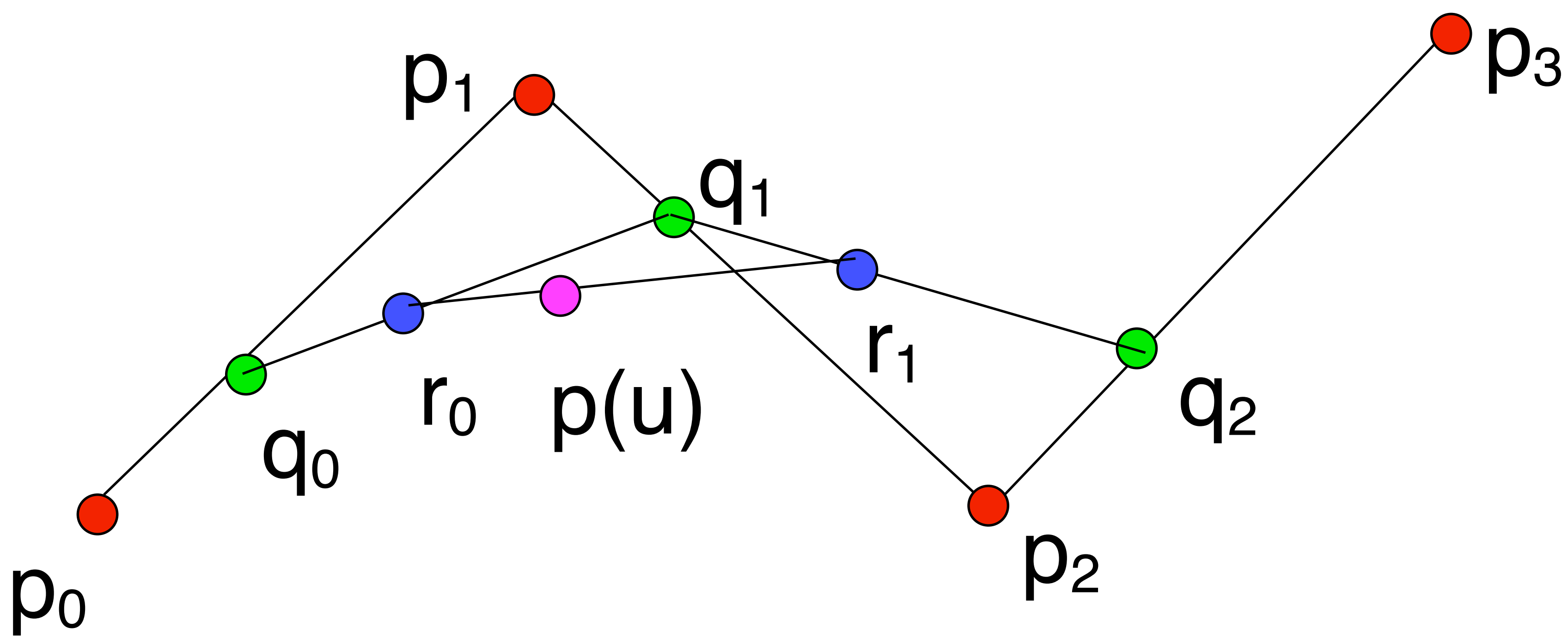
# de Casteljau's algorithm

**A Bézier is really an *interpolation of interpolations*!**



# de Casteljau's algorithm

## Linear interpoations of linear interpolations until only one point remains



# de Casteljau's algorithm

**Gives us the Bernstein polynomials of any level we want.**

**Linear (2 points) = plain interpolation**
**Quadratic Béziers (3 points)**
**Cubic (4 points)**
**Higher levels possible but not practical**

# de Casteljau's algorithm

## Obvious from figure/method:

- **Bézier is always inside convex hull**

- **Fit together sections by keeping points along a line also obvious - we must start along the tangent!**

# Drawing splines

**Subdivide the spline until the error is small enough.**