Interpolating Splines

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Spline Interpolation

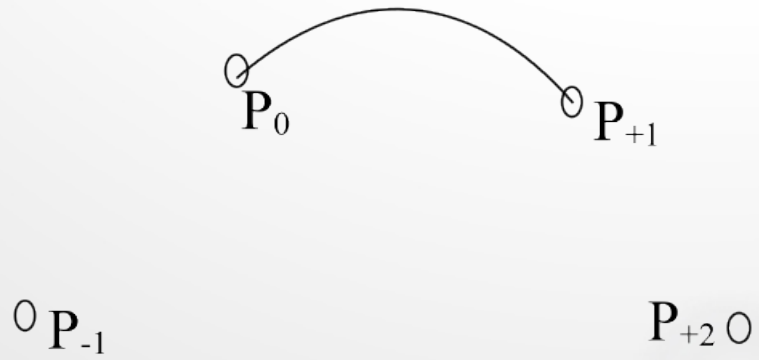
- **Common problem: need a smooth interpolation**
- **smooth = higher continuity than c0**
  - c1 enough for most purposes, e.g. animation
  - camera movement: g2 might be better
- **Other characteristics**
  - **affine invariant:** all blend weights always add up to one
  - **local control:** each control-point influences only a limited number of calculated points

Spline Interpolation

- **Here:**
  - only polynomial of 3rd degree
  - affine invariant, local control
  - c1 continuous

Cardinal Spline

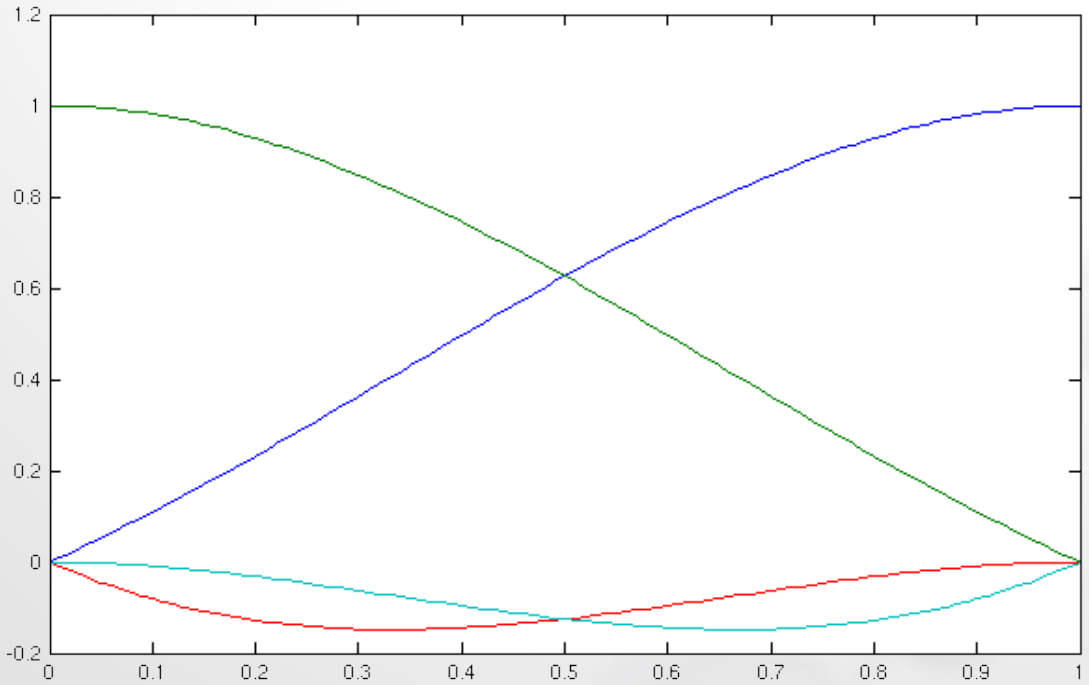
- Specified by control points
- Shape can be varied by a tension parameter t
- Calculated from 4 control points, define the curve between the middle two



- Even called Catmull-Rom (for t=0?)

Cardinal Splines

$$(-\alpha u^3 + 2\alpha u^2 - \alpha u)\, P_{-1}$$
$$((2-\alpha)u^3 + (\alpha-3)u^2 + 1)\, P_0$$
$$((\alpha-2)u^3 + (3-2\alpha)u^2 + \alpha u)\, P_{+1}$$
$$(\alpha u^3 - \alpha u^2)\, P_{+2}$$



Catmull-Rom Spline ($\alpha = 0.5$)

$$P(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} -1/2 & 3/2 & -3/2 & 1/2 \\ 1 & -5/2 & 2 & -1/2 \\ -1/2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{k-1} \\ p_k \\ p_{k+1} \\ p_{k+2} \end{bmatrix}$$

$$\begin{aligned} P(u) = \quad & p_{k-1}\,(-u^3/2 + u^2 - u/2) + \\ & p_k\,(3u^3/2 - 5u^2/2 + 1) + \\ & p_{k+1}\,(-3u^3/2 + 2u^2 + u/2) + \\ & p_{k+2}\,(u^3/2 - u^2/2) \end{aligned}$$

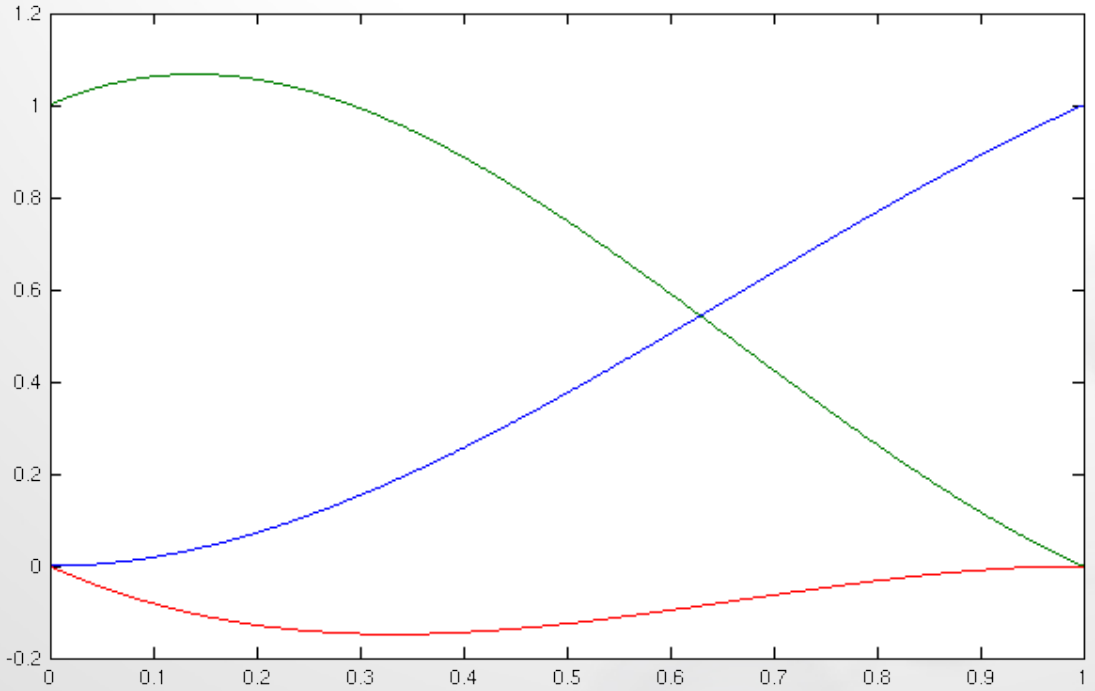
$$= \quad p_{k-1}*CAR_0(u) + p_k*CAR_1(u) + p_{k+1}*CAR_2(u) + p_{k+2}*CAR_3(u)$$

Ogniewski (?) Splines

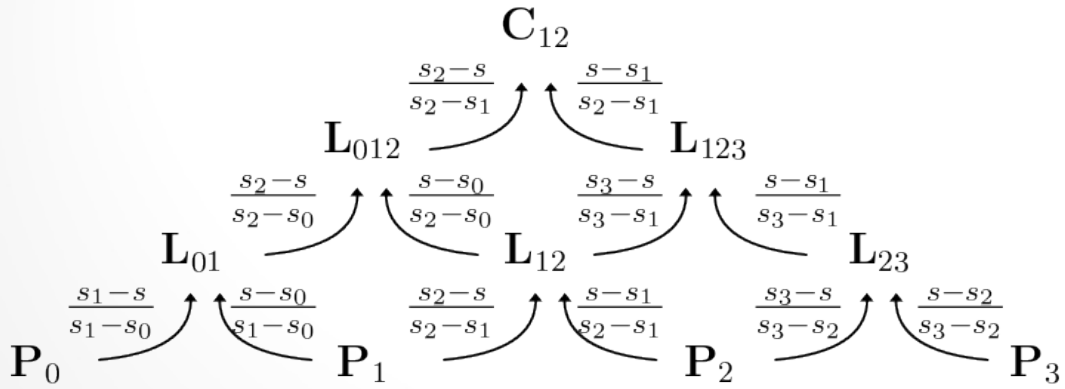
- Realtime applications: future points not known
- Could be extrapolated, but will lead to discontinuities
- Can we do interpolation without future points?

Ogniewski (?) Splines

$$(-\alpha u^3 + 2\alpha u^2 - \alpha u)\, P_{-1}$$
$$(2u^3 + (-3-\alpha)u^2 + \alpha u + 1)\, P_0$$
$$((\alpha-2)u^3 + (3-\alpha)u^2)\, P_{+1}$$



Cardinal Spline: centripetal parameterization



$$S_0 = 0,$$
$$S_{i+1} = |P_i - P_{i+1}|^{0.5} + s_i$$

Cem Yuksel, Scott Schaefer, John Keyser: Parameterization and applications of Catmull-Rom curves, 2011

Conclusion

- **Object representation**
  - Polyhedra
  - Quadric
  - Constructive Geometry
- **Splines**
  - Approximation spline (mostly used for Object representation)
  - Interpolating spline (mostly used for Animation)
- **Don't forget: the important think is how the result looks, not the method!**

Thank you very much!
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